ELECTRO MAGNETIC FIELDS

CONTENTS

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Electro Magnetic Fields

1. **Introduction:**

In communication systems, circuit theory is valid at both the transmitting end as well as the receiving end but it fails to explain the flow between the transmitter and receiver.

Circuit theory deals with only two variables that is voltage and current whereas Electromagnetic theory deals with many variables like electric field intensity, magnetic field intensity etc.,

Mostly three space variables are involved in electromagnetic field problems. Hence the solution becomes complex. For solving field problems we need strong background of vector analysis.

Maxwell has applied vectors to Gauss's law, Biot Savart's law, Ampere's Law and Faraday's Law. His application of vectors to basic laws, produced a subject called "Field Theory".

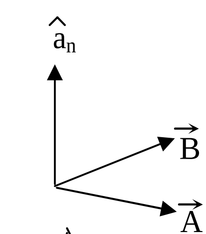
2. Scalar and Vector Products

a) Dot Product: is also called scalar product. Let 'θ' be the angle between vectors A and B.

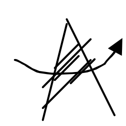
$$\boxed{\vec{A} \cdot \vec{B} = |A|\,|B|\cos\theta}$$

The result of dot product is a scalar. Dot product of force and distance gives work done (or) Energy which is scalar.

b) Cross product: is also called vector product.



$$\boxed{\vec{A} \times \vec{B} = |A|\,|B| \sin\theta\, \hat{a}_n}$$



$\vec{S} = |S|\, \hat{a}_n$ where $|S| = |A|\,|B| \sin\theta$

To find the direction of S, consider a right threaded screw being rotated from A to B. i.e. perpendicular to the plane containing the vectors A and B.

$$\therefore\ \vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$$

3. Operator Del (∇):

Del is a vector three dimensional partial differential operator. It is defined in Cartesian system as

$$\boxed{\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}}$$

Del is a very important operator. There are 3 possible operations with del. They are gradient, divergence and curl.

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4. GRADIENT:

Gradient is a basic operation of a Del operator that can operate only on a scalar function. Consider a scalar function 't'. The gradient of 't' can be mathematically defined and symbolically expressed as below.

$$\nabla t \; (\text{Grad } t) = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} \right) t$$

$$\boxed{\nabla t = \frac{\partial t}{\partial x}\hat{i} + \frac{\partial t}{\partial y}\hat{j} + \frac{\partial t}{\partial z}\hat{k}}$$

→ Vector

Gradient of scalar function is a vector function.

Ex:- Temperature of soldering iron is scalar, but rate of change of temperature is a Vector. In a cable, potential is scalar. The rate of change of potential is a vector (Electric field intensity).

5. DIVERGENCE:-

Divergence is a basic operation of the Del operator which can operate only on a vector function through a dot product.

Considering a vector function $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

The divergence of vector A mathematically and symbolically expressed as shown below.

$$\nabla.\vec{A} \; (\text{Div } A) = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} \right) \cdot \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \right)$$

$$\boxed{\nabla.\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}}$$

→ Scalar

Divergence of vector function is a scalar function.

Let D = flux density vector
D.ds = flux through the surface ds
The flux through the entire surface is $\iint_s D.ds$

Note: Divergence of D gives net outflow of flux per unit volume.

$$\therefore \boxed{\nabla.\vec{D} = \underset{\Delta V \to 0}{\text{Lt}} \frac{\iint_s D.ds}{\Delta V}}$$

6. CURL:

Curl is a basic operation of a Del operator which can perform only on a vector function through a cross product.

$$\nabla \times \vec{A} \ (\text{Curl } A) = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \times \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

, vector

$$= \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{i} - \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right)\hat{j} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{k}$$

Curl of a vector function is a vector function.
Curl deals with rotation.
If the curl of a vector field vanishes, it is called Irrotational field.
Curl is mathematically defined as circulation per unit area.

$$\text{Curl } v = \frac{\text{circulation}}{\text{UnitArea}}$$

$$\therefore \boxed{\text{Curl } \vec{v} = \underset{\Delta s \to 0}{\text{Lt}} \frac{\oint v \cdot dl}{\Delta s}}$$

7. Laplacian of a Scalar function (t) :-

Double operation

$$\nabla \cdot (\nabla t) = \nabla^2 t = \Delta t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

$$\boxed{\nabla^2 t = \underbrace{\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)}_{\text{Laplacian operator}} t}$$

Laplacian of a scalar function is a scalar function.

8. Laplacian of a Vector function ($\vec{A}$):

$$\text{Let } \vec{A} = A_x i + A_y j + A_z k$$

$$\boxed{\nabla^2\vec{A} = \left(\frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2}\right)\hat{i} + \left(\frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_y}{\partial z^2}\right)\hat{j} + \left(\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2}\right)\hat{k}}$$

Laplacian of a vector function is a vector function.

9. Concept of field:

Considering a region where every point is associated with a function, then the region is said to have a field.

If associated function is a scalar then it is a scalar field and if the associated function is a vector function then it is a vector field.

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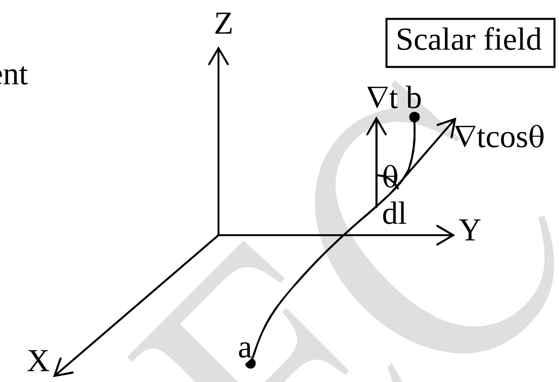
10. Basic types of vector fields:

- **a)** Solenoidal vector field ($\nabla\cdot\vec{A} = 0$)
- **b)** Irrotational vector field ($\nabla \times \vec{A} = 0$)
- **c)** Vector fields that are both solenoidal & irrotational
- **d)** Vector fields which are neither solenoidal nor irrotational

11. Fundamental theorem of Gradient:

Statement: consider an open path from ‘a’ to ‘b’ in a scalar field as shown. The line integral of the tangential component of the gradient of a scalar function along the open path is equal to path the effective value of the associated scalar function at the boundaries of the open path.



If ‘t’ is the associated scalar function, then according to the fundamental theorem of gradient

$$\boxed{\int_a^b (\nabla t)\cdot\vec{dl} = t(b) - t(a)}$$

Corollary-1:

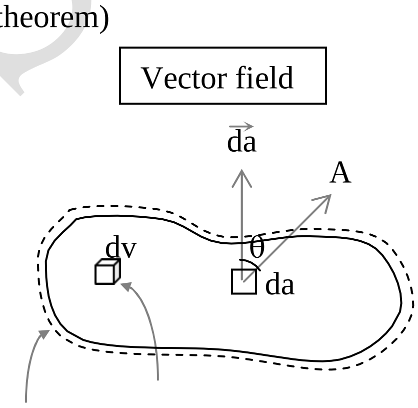
If it is a closed path in scalar field, then

$$\boxed{\oint_a^b (\nabla t)\cdot\vec{dl} = 0}$$

Corollary-2:

A line integral $\int_a^b (\nabla t)\cdot\vec{dl}$ is independent of the open path.

12. Fundamental theorem of Divergence:- (Gauss theorem)



Statement:

Consider a closed surface in vector field. The volume integral of the divergence of the associated vector function carried within a enclosed volume is equal to the surface integral of the normal component of the associated vector function carried over an enclosing surface.

If associated vector function is $\vec{A}$, then according to fundamental theorem of divergence,

$$\boxed{\iiint_V (\nabla\cdot\vec{A})dv = \oiint_S \vec{A}\cdot\vec{da}}$$

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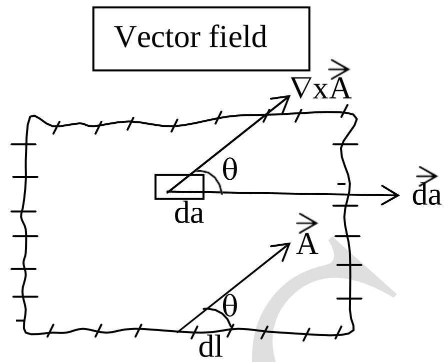
Note: Area vector is always outward normal

13. Fundamental theorem of Curl:- (Stokes theorem)

Statement: Considering an open surface placed in a vector field, the surface integral of the normal component of the curl of the associated vector function carried over the open surface is equal to the line integral of the tangential component of the associated vector function along the boundary of the open surface.

If associated vector function is $\vec{A}$, then

$$\boxed{\iint_s (\nabla \times \vec{A}) \cdot \vec{da} = \oint \vec{A} \cdot \vec{dl}}$$



Corollary-1: If it is a closed surface

$$\boxed{\oiint_S (\nabla \times \vec{A}) \cdot \vec{da} = 0}$$

Since there is no boundary and hence

$$\boxed{\oint \vec{A} \cdot \vec{dl} = 0}$$

Corollary-2: $\oint \vec{A} \cdot \vec{dl}$ is constant for a fixed boundary.Therefore , $\iint_s (\nabla \times \vec{A}) \cdot \vec{da}$ is independent of the type of open surface.

14. Vector Identities:

a) $\nabla \times \nabla \phi = 0$

b) $\nabla \cdot \nabla \times \vec{A} = 0$

c) $\nabla \cdot \phi \vec{A} = \nabla \phi \cdot \vec{A} + \phi (\nabla \cdot \vec{A})$

d) $\nabla \times \phi \vec{A} = \nabla \phi \times \vec{A} + \phi (\nabla \times \vec{A})$

e) $\nabla \times \nabla \times \vec{A} = \nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

f) $\nabla \cdot \nabla \phi = \nabla^2 \phi$

g) $\nabla(\phi F) = \phi(\nabla \cdot F) + F \nabla \phi$

h) Div (u x v) = v curl u – u curl v

i) $\vec{A} \cdot \vec{B} \times \vec{C} = \vec{B} \cdot \vec{C} \times \vec{A} = \vec{C} \cdot \vec{A} \times \vec{B}$

j) $\nabla \cdot \vec{A} \times \vec{B} = \vec{B} \cdot \nabla \times \vec{A} - \vec{A} \cdot \nabla \times \vec{B}$

k) $\nabla^2 \vec{A} = \nabla (\nabla \cdot \vec{A}) - \nabla \times (\nabla \times \vec{A})$

15. Co-ordinate systems:

a) Cartesian co-ordinate system (x,y,z)
b) Spherical co-ordinate system (r,θ,ϕ)
c) Cylindrical co-ordinate system (r,ϕ,z)

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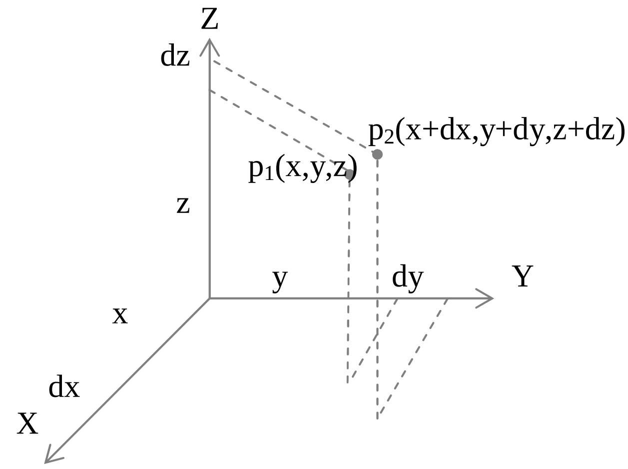
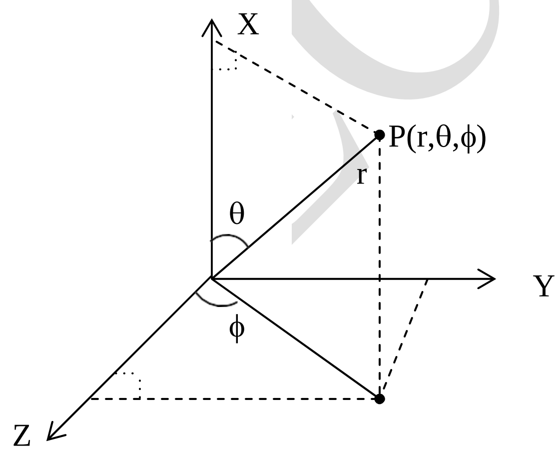
15.a) Cartesian co-ordinate system :- (x,y,z)



Differential length, $\boxed{\vec{dl} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}}$

15.b) Spherical co-ordinate system :- (r,θ,ϕ)



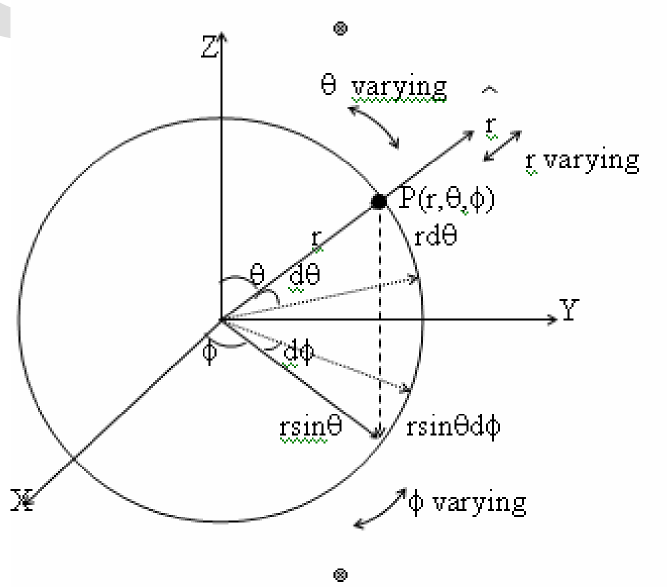
Cartesian to spherical

$X = r\sin\theta\cos\phi$
$Y = r\sin\theta\sin\phi$
$Z = r\cos\theta$

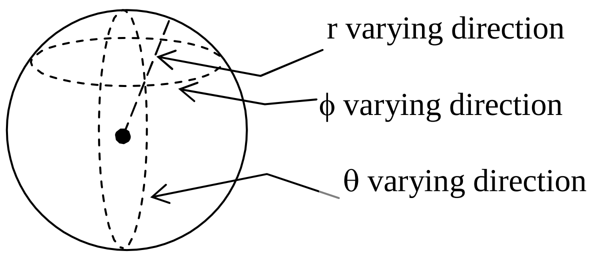
Spherical to Cartesian

$r = \sqrt{x^2 + y^2 + z^2}$
$\theta = \cos^{-1}\left(\frac{Z}{\sqrt{x^2 + y^2 + z^2}}\right)$
$\phi = \tan^{-1}(y/x)$

Note: In Spherical system unit vectors are $\hat{r}, \hat{\theta}, \hat{\phi}$
Differential Length Vector:

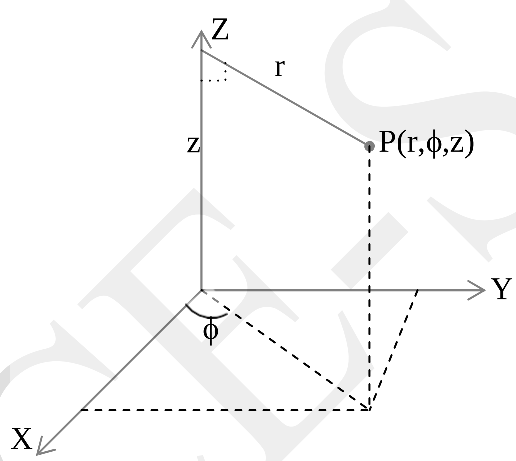


Ranges: $r = 0 \rightarrow \propto$
$\theta = 0 \rightarrow \Pi$
$\phi = 0 \rightarrow 2\Pi$



Differential length, $\boxed{\vec{dl} = (dr)\hat{r} + (rd\theta)\hat{\theta} + (r\sin\theta\, d\phi)\,\hat{\phi}}$

15.c) Cylindrical co-ordinate system: (r, ϕ, z)



Cartesian to Cylindrical

$r = \sqrt{x^2 + y^2}$
$\phi = \tan^{-1}(y/x)$
$z = z$

Cylindrical to Cartesian

$x = r\cos\phi$
$y = r\sin\phi$
$z = z$

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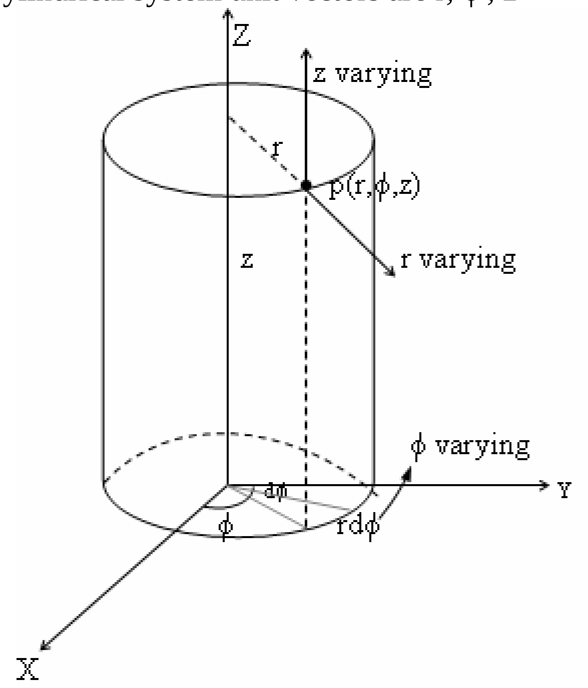
Note: In cylindrical system unit vectors are $\hat{r}, \hat{\phi}, \hat{z}$



Differential Length Vector:

Ranges:

$r = 0 \rightarrow \infty$

$\phi = 0 \rightarrow 2\Pi$

$z = -\infty \rightarrow +\infty$

Differential length, $\boxed{\vec{dl} = (dr)\,\hat{r} + (rd\phi)\,\hat{\phi} + (dz)\,\hat{z}}$

16. Differential areas: ($\vec{da}$ (or) $\vec{ds}$)

a) Cartesian system:

$$\vec{dl} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$\boxed{\vec{da} = dx\,dy\,\hat{k}}$$

b) Spherical system:

$$\vec{dl} = (dr)\,\hat{r} + (rd\theta)\,\hat{\theta} + (r\sin\theta\,d\phi)\,\hat{\phi}$$

$$\boxed{\vec{da} = (r^2 \sin\theta\,d\theta\,d\phi)\,\hat{r}}$$

c) Cylindrical system:

$$\vec{dl} = (dr)\,\hat{r} + (rd\phi)\,\hat{\phi} + (dz)\,\hat{z}$$

$$\boxed{\vec{da} = (rd\phi\,dz)\,\hat{r}}$$

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17. Differential volumes: (dv)

a) Cartesian system:

$$\vec{dl} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$\boxed{dv = dx\,dy\,dz}$$

b) Spherical system:

$$\vec{dl} = (dr)\,\hat{r} + (r d\theta)\,\hat{\theta} + (r \sin\theta\, d\phi)\,\hat{\phi}$$

$$\boxed{dv = r^2 \sin\theta\, dr\, d\theta\, d\phi}$$

c) Cylindrical system:

$$\vec{dl} = (dr)\,\hat{r} + (r d\phi)\,\hat{\phi} + (dz)\,\hat{z}$$

$$\boxed{dv = r dr\, d\phi\, dz}$$

18. Dot Product between Spherical & Cartesian system unit vectors.

Spherical \ Cartesian	$\hat{i}$	$\hat{j}$	$\hat{k}$
r	sinθ cosϕ	sinθ sinϕ	cosθ
θ	cosθ cosϕ	cosθ sinϕ	- sinθ
$\hat{\phi}$	- sinϕ	cosϕ	0

19. Dot Product between Cylindrical & Cartesian system unit vectors.

Cylindrical \ Cartesian	$\hat{i}$	$\hat{j}$	$\hat{k}$
$\hat{r}$	cosϕ	sinϕ□0	
$\hat{\phi}$	- sinϕ	cosϕ□0	
$\hat{z}$	0	0	1

20. General Curvilinear Co-ordinate System

Let h_1, h_2 & h_3 be scale factors
u_1, u_2 & u_3 be co-ordinate system

$\hat{e}_1$, $\hat{e}_2$ & $\hat{e}_3$ be unit vectors

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Cartesian system	Spherical system	Cylindrical system
$h, h_2, h_3 \equiv 1,1,1$	$h, h_2, h_3 \equiv 1, r, r\sin\theta$	$h, h_2, h_3 \equiv 1, r, 1$
$\hat{e}_1, \hat{e}_2, \hat{e}_3 \equiv \hat{i}, \hat{j}, \hat{k}$	$\hat{e}_1, \hat{e}_2, \hat{e}_3 \equiv \hat{r}, \hat{\theta}, \hat{\phi}$	$\hat{e}_1, \hat{e}_2, \hat{e}_3 \equiv \hat{r}, \hat{\phi}, \hat{z}$
$u_1, u_2, u_3 \equiv x, y, z$	$u_1, u_2, u_3 \equiv r, \theta, \phi$	$u_1, u_2, u_3 \equiv r, \phi, z$

In General:

1) $$\nabla t = \frac{1}{h_1}\frac{\partial t}{\partial u_1}\hat{e}_1 + \frac{1}{h_2}\frac{\partial t}{\partial u_2}\hat{e}_2 + \frac{1}{h_3}\frac{\partial t}{\partial u_3}\hat{e}_3$$

2) $$\nabla \cdot \vec{A} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}(A_1h_2h_3) + \frac{\partial}{\partial u_2}(A_2h_3h_1) + \frac{\partial}{\partial u_3}(A_3h_1h_2)\right]$$

3) $$\nabla \times \vec{A} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\hat{e}_1 & h_2\hat{e}_2 & h_3\hat{e}_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ A_1h_1 & A_2h_2 & A_3h_3 \end{vmatrix}$$

4) $$\nabla^2 t = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}\left\{\frac{h_2h_3}{h_1}\left(\frac{\partial t}{\partial u_1}\right)\right\} + \frac{\partial}{\partial u_2}\left\{\frac{h_3h_1}{h_2}\left(\frac{\partial t}{\partial u_2}\right)\right\} + \frac{\partial}{\partial u_3}\left\{\frac{h_1h_2}{h_3}\left(\frac{\partial t}{\partial u_3}\right)\right\}\right]$$

Let: $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k} \longrightarrow$ Cartesian system

$= A_1\hat{e}_r + A_2\hat{e}_\theta + A_3\hat{e}_\phi \longrightarrow$ Spherical system

$= A_1 e_r + A_2 e_\phi + A_3 e_z \longrightarrow$ Cylindrical

In Cartesian system:

1) $$\nabla t = \frac{\partial t}{\partial x}\hat{i} + \frac{\partial t}{\partial y}\hat{j} + \frac{\partial t}{\partial z}\hat{k}$$

2) $$\nabla \cdot \vec{A} = \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}$$

3) $$\nabla \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$$

4) $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

In Spherical system:

1) $$\nabla t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

2) $$\nabla \cdot \vec{A} = \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}(A_1 r^2\sin\theta) + \frac{\partial}{\partial \theta}(A_2 r\sin\theta) + \frac{\partial}{\partial \phi}(A_3 r)\right]$$

3) $$\nabla \times \vec{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A1 & rA_2 & r\sin\theta A_3 \end{vmatrix}$$

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4) $\nabla^2 t = \dfrac{1}{r^2 \sin\theta}\left[\dfrac{\partial}{\partial r}\left\{r^2 \sin\theta\left(\dfrac{\partial t}{\partial r}\right)\right\} + \dfrac{\partial}{\partial \theta}\left\{\dfrac{r \sin\theta}{r}\left(\dfrac{\partial t}{\partial \theta}\right)\right\} + \dfrac{\partial}{\partial \phi}\left\{\dfrac{r}{r \sin\theta}\left(\dfrac{\partial t}{\partial \phi}\right)\right\}\right]$

In Cylindrical system:

1) $\nabla t = \dfrac{\partial t}{\partial r}\hat{r} + \dfrac{1}{r}\dfrac{\partial t}{\partial \phi}\hat{\phi} + \dfrac{\partial t}{\partial z}\hat{z}$

2) $\nabla \cdot \vec{A} = \dfrac{1}{r}\left[\dfrac{\partial}{\partial r}(A_1 r) + \dfrac{\partial}{\partial \phi}A_2 + \dfrac{\partial}{\partial z}(A_3 r)\right]$

3) $\nabla \times \vec{A} = \dfrac{1}{r}\begin{vmatrix} \hat{r} & r\hat{\phi} & \hat{z} \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \phi} & \dfrac{\partial}{\partial z} \\ A_1 & rA_2 & A_3 \end{vmatrix}$

4) $\nabla^2 t = \dfrac{1}{r}\left[\dfrac{\partial}{\partial r}\left\{r\left(\dfrac{\partial t}{\partial r}\right)\right\} + \dfrac{\partial}{\partial \phi}\left\{\dfrac{1}{r}\left(\dfrac{\partial t}{\partial \phi}\right)\right\} + \dfrac{\partial}{\partial z}\left\{r\left(\dfrac{\partial t}{\partial z}\right)\right\}\right]$

OBJECTIVES

One Mark Questions

1) If the vectors $\vec{A}$ and $\vec{B}$ are conservative then (Engg.Services,1993)
a) $\vec{A} \times \vec{B}$ is solenoidal   b) $\vec{A} \times \vec{B}$ is conservative
c) $\vec{A} + \vec{B}$ is solenoidal   d) $\vec{A} - \vec{B}$ is solenoidal

2) The value of $\oint d.l$ along a circular radius of 2 units is (IES, 93)
a) zero   b) 2Π c) 4Π d) 8Π

3) which of the following relations is correct? (BEL, 95)
a) $\nabla \times (AB) = \nabla A \times B - A.\nabla B$ b) $\nabla \cdot (AB) = \nabla A.B + A.\nabla B$
c) $\nabla (AB) = A.\nabla B + B.\nabla A$ d) all the three

4) $\nabla \cdot (\nabla \times A)$ is equal to (BEL, 95)
a) 0   b) 1   c) ∞ d) none of these

5) Given points A(2,3,-1) and B(4, -50^0,2) find the distance from A to B
a) 3.74   b) 4.47   c) 16.7   d) 6.79

6) Find the nature of the given vector field defined by $\vec{A} = 30\hat{i} - 2xy\hat{j} + 5xz^2\hat{k}$
a) Neither Solinoidal nor irrotational   b) Solinoidal & irrotational
c) Only Solinoidal   d) Only irrotational

7) Find the nature of given vector field defined by $\vec{A} = yz\hat{i} + zx\hat{j} + xy\hat{k}$
a) Neither Solinoidal nor irrotational   b) Solinoidal & irrotational
c) Only Solinoidal   d) Only irrotational

8) A vector field is given by $\vec{A} = 3xy\hat{i} - y^2\hat{j}$. Find $\int_c \vec{A}.\vec{dl}$ where 'c' is the curve $y = 2x^2$ in the x-y plane from (0,0) to (1,2)
a) -9/2   b) 7/6   c) -7/6   d) 2/3

9) Find the laplacian of the scalar function $v = (\cos\phi)/r$ (cylindrical system).
a) 5   b) 0   c) 7/6   d) 8

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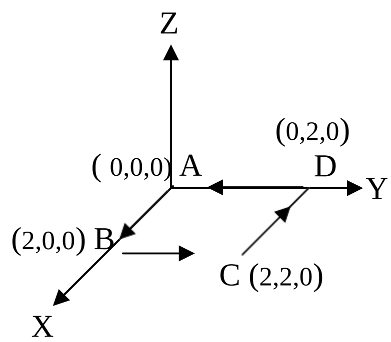
Two mark Questions

10) Find $\nabla\,(1/r)$, where $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$

a) $\dfrac{\hat{r}}{r^2}$ b) 0 c) $\dfrac{-\hat{r}}{r^2}$ d) $r^2\,\hat{r}$

11) Find the line integral of the vector function $\vec{A} = x\,\hat{i} + x^2 y\,\hat{j} + y^2 x\,\hat{k}$ around a square contour ABCD in the x-y plane as shown.



a) 0 b) 10 c) -1 d) 8

12) For the vector function $\vec{A} = xy^2\,\hat{i} + yz^2\,\hat{j} + 2xz\,\hat{k}$, calculate $\int_c \vec{A}\,.\,\vec{dl}$ Where c is the straight line joining points (0,0,0) to (1,2,3)

a) 2π b) 8π c) 16 d) 13

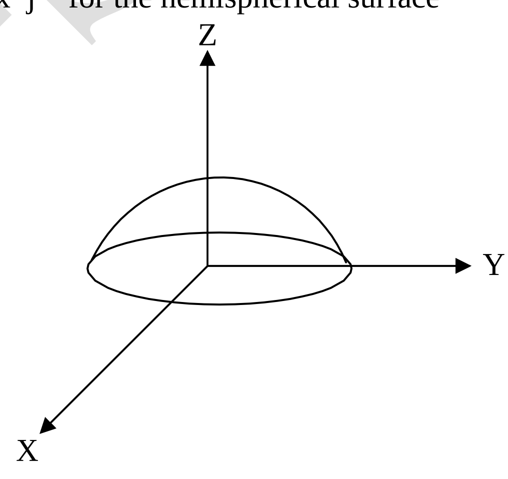
13) A Circle of radius 2 units is centered at the origin and lies on the YZ-plane. If $\vec{A} = 3y^2\,\hat{i} + 4z\,\hat{j} + 6y\,\hat{k}$, find the line integral $\int_c \vec{A}\,.\vec{dl}$. Where C is the circumference of the circle.

a) π b) 8π c) 0 d) $\pi/3$

14) Represent point P (0,1,1)m given in Cartesian co-ordinate system, in spherical co-ordinates .

a) (1, $\pi/3$, π) b) ($\sqrt{2}$, $\pi/4$, π) c) ($\sqrt{2}$, $-\pi/4$, π) d) ($\sqrt{2}$, $\pi/4$, $-\pi$)

15) Find $\iint_s (\nabla \times \vec{A})\,.\,\vec{da}$ where $\vec{A} = y\,\hat{i} - x\,\hat{j}$ for the hemispherical surface $x^2 + y^2 + z^2 = b^2$; $z \geq 0$



a) $-2\pi b^2$

b) 2π

c) $-2\pi b$

d) $2\pi b^2$

Key: 1) a 2) a 3) d 4) a 5) d 6) a 7) b 8) c 9) b 10) c 11) d 12) c 13) b 14) b 15) a

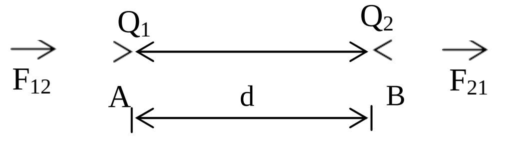
ECE-SAEC

TOPIC – 2 : ELECTRIC FIELD INTENSITY E M F

Electrostatics is a science that deals with the charges at rest. Static charges produce electric field.

In electromagnetic theory there is a fundamental problem with regard to the force between the electric charges. Let us start our study with an introduction of coulomb's law

Coulomb's Law:



This law states that considering two point charges separated by a distance, the force of attraction (or) repulsion is directly proportional to the product of the magnitudes and inversely proportional to the square of the distance between them.

$$F \propto \frac{|Q_1|\,|Q_2|}{d^2}$$

$$F = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{|Q_1|\,|Q_2|}{d^2}$$

Force acting on Q1 due to Q2, $\vec{F}_{12} = \dfrac{|Q_1|\,|Q_2|}{4\pi\epsilon_0 d^2}\ \widehat{BA}$

Force acting on Q2 due to Q1, $\vec{F}_{21} = \dfrac{|Q_1|\,|Q_2|}{4\pi\epsilon_0 d^2}\ \widehat{AB}$

This law is an imperial law and difficult to understand how exactly a force is communicated between them. Michel Faraday gives a satisfactory explanation of coulomb's law by introducing the concept of electric field.

According to Faraday, Q1 experiences a force because it is placed in the electric field of Q2. And Q2 experiences a force because it is placed in the electric field of Q1.

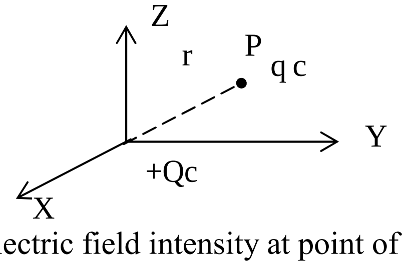
Concept Of Electric Field:

An electric field is said to exist at a particular point, if a test charge placed at that point experiences a force.

If 'q' is the test charge and $\vec{F}$ is the force experienced by the test charge, then the force per unit test charge is known as Electric field intensity. Expressed in N/C or V/m

$$\boxed{\vec{E} = \frac{\vec{F}}{q}} \quad \text{N/C (or) V/M}$$

ELECTRIC FIELD DUE TO A POINT CHARGE:



Consider a point charge of '+Q' c at origin. In order to find electric field intensity at point of observation P, consider a Unit test Charge 'q' c at P.

(Contd …13)

ECE-SAEC

Therefore, the force experienced by the test charge is

$$\vec{F} = \frac{|Q_1|\, q}{4\pi\epsilon_0 r^2}\,\hat{r}$$

We know, $$\vec{E} = \frac{\vec{F}}{q}$$

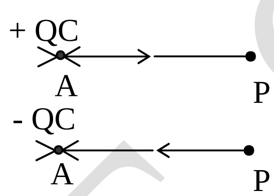
$$\boxed{\therefore \vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{r}}$$

NOTE: Thus electric field intensity is independent of the amount of test charge.
In Cartesian system:

$$\vec{F} = \frac{Q}{4\pi\epsilon_0 r^2} \cdot \frac{\vec{r}}{|\,r\,|}$$

$$\boxed{\vec{F} = \frac{Q}{4\pi\epsilon_0 (x^2+y^2+z^2)^{3/2}}\,(x\,\hat{i} + y\,\hat{j} + z\,\hat{k})}$$

ELECTRIC FIELD DUE TO A POINT CHARGE LOCATED AT ANY GENERAL POSITION:



$$\therefore \vec{E} = \frac{Q}{4\pi\epsilon_0 (AP)^2}\,\widehat{AP}$$

$$\therefore \vec{E} = \frac{Q}{4\pi\epsilon_0 (PA)^2}\,\widehat{PA}$$

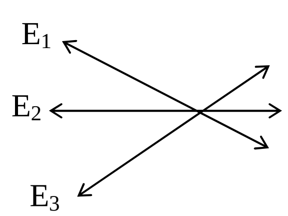
Electric field is always directed away from the point charge towards the point of observation(P), if it is a positive charge.

Similarly, electric field is directed away from the point of observation towards the point charge, if it is a negative charge.

PRINCIPLE OF SUPERPOSITION:

The principle of superposition says that electric field due to any charge is unaffected by the presence of other charges.

In a system of discrete charges the net electric field is obtained by the vectorically adding up the individual electric fields.



Net electric field intensity $\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3} + \ldots\ldots..$

(Contd …14)

ECE-SAEC

Electric field due to continuous charges distribution:

Continuous charge distribution is categorized into 3 types.

a) Line charge distribution:

If the charge is continuously distributed along the line with line charge density "ρ_L" c/m, it is called line charge distribution.

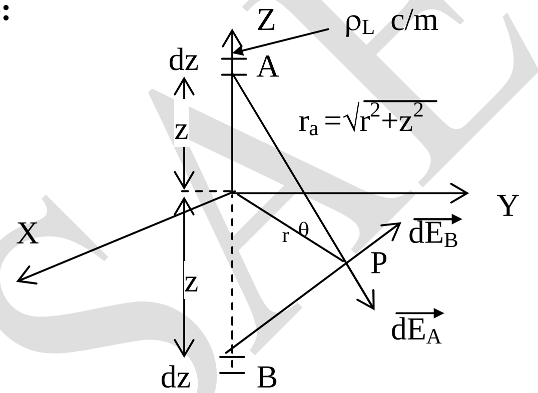
b) Surface Charge Distribution:

If the charge is continuously distributed over a surface with surface charge density "ρ_s " c/m^2, it is called surface charge distribution.

c) Volume Charge Distribution:

If the charge is continuously distributed over a volume with volume charge density "ρ_v " c/m^3, it is called volume charge distribution.

Electric field due to an infinite line charge:



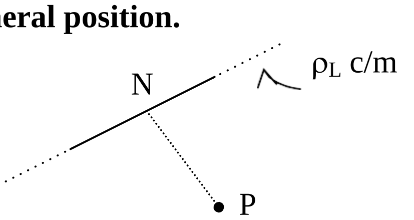
Consider an infinite line charge with a line charge density ρ_L c/m placed along the z-aixs. Let the point of observation 'P' be on x-y plane.

Net electric field at P, $\boxed{\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 r}\,\hat{r}}$

Electric field due to infinite Line charge located at any general position.

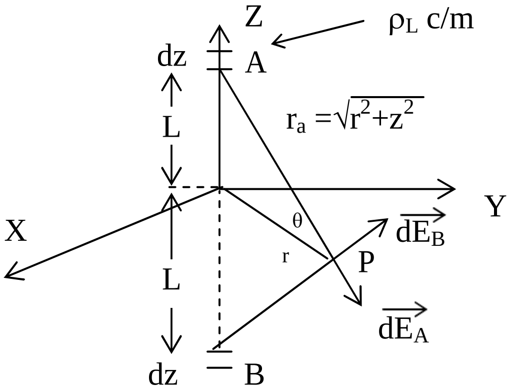
i) $\vec{E}_{\text{at P}} = \frac{\rho_L}{2\pi\epsilon_0 NP}\,\widehat{NP}$, if it is a +ve line charge.

ii) $\vec{E}_{\text{at P}} = \frac{\rho_L}{2\pi\epsilon_0 NP}\,\widehat{PN}$, if it is a -ve line charge.



Electric field due to a finite Line charge(2L) along the perpendicular bisector.

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 r}\,\frac{L}{\sqrt{L^2 + r^2}}\,\hat{r}}$$



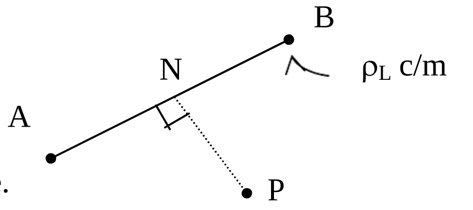
(Contd ...15)

ECE-SAEC

Electric field due to a finite line charge located at any general position.

i) $\vec{E}_{\text{at P}} = \dfrac{\rho_L}{2\pi\epsilon_0 NP} \cdot \dfrac{BN}{\sqrt{BN^2 + NP^2}} \; \hat{NP}$, if it is a +ve line charge.

ii) $\vec{E}_{\text{at P}} = \dfrac{\rho_L}{2\pi\epsilon_0 NP} \cdot \dfrac{BN}{\sqrt{BN^2 + NP^2}} \; \hat{PN}$, if it is a -ve line charge.



Electric field due to a finite line charge

(OA ≠ OB)

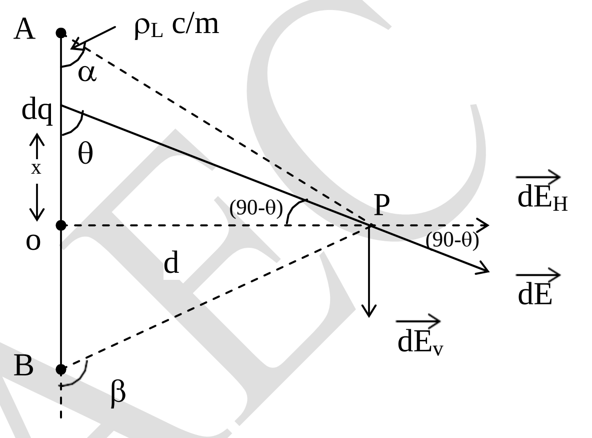
i) $\vec{E}_H = \dfrac{\rho_L}{4\pi\epsilon_0 d}(\cos\alpha - \cos\beta)$

ii) $\vec{E}_V = \dfrac{\rho_L}{4\pi\epsilon_0 d}(\sin\beta - \sin\alpha)$

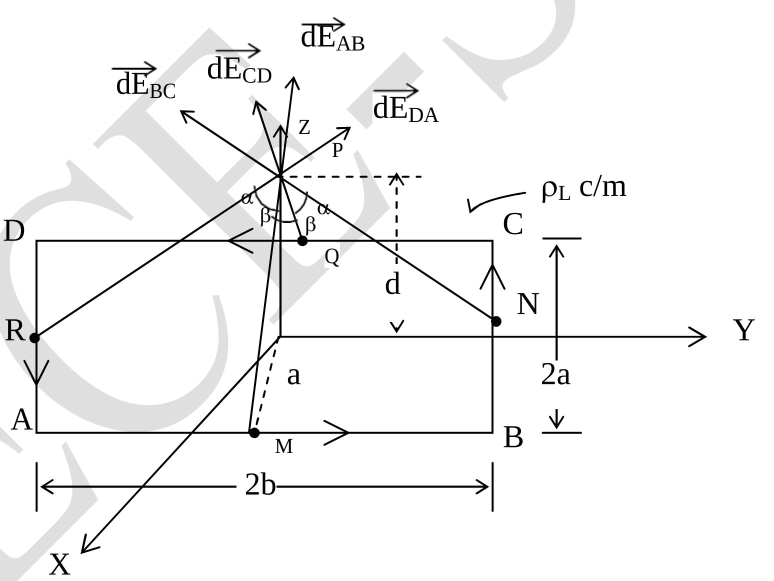
iii) Net electric field intensity, $\vec{E} = \sqrt{E_H^2 + E_V^2}$

iv) If 'O' is the mid point, β=(180-α). As line tends to infinity, α ®0, β® π $E_v = 0$

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 d}\,\hat{op}}$$



Electric field due to Rectangular line charge along it axis.



$\vec{E}_{\text{at P}} = \vec{E}_{AB} + \vec{E}_{CD} + \vec{E}_{BC} + \vec{E}_{DA}$

$= 2\,[\,\vec{E}_{AB} + \vec{E}_{BC}\,]$

$$\boxed{E = \frac{\rho_L d}{\pi\varepsilon_0\sqrt{a^2+b^2+d^2}} \cdot \left(\frac{a}{b^2+d^2} + \frac{b}{a^2+d^2}\right)\hat{K}}$$

(Contd …16)

ECE-SAEC

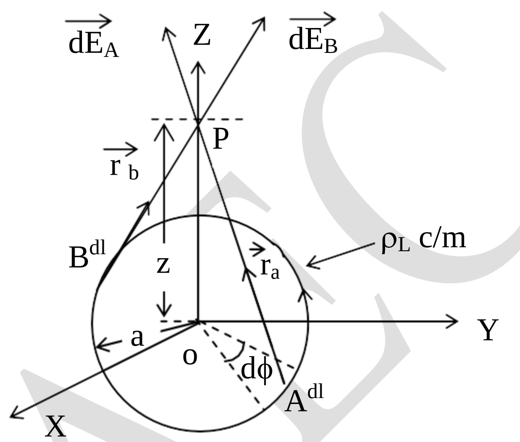
Corollary:1 If it is a square line charge a=b

$$\boxed{\vec{E} = \frac{2\rho_L da}{\pi\varepsilon_o\sqrt{2a^2+d^2}\,.\,(a^2+d^2)} . \hat{K}}$$

Corollary:2 If d=0 i.e. the electric field at orgin

$$\boxed{\vec{E} = 0}$$

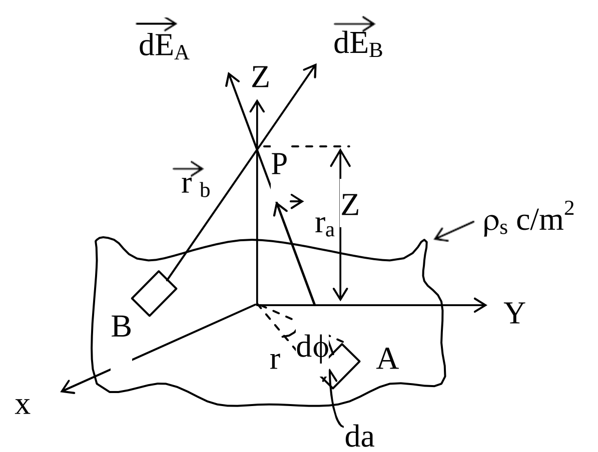
Electric field due to a circular line charge along its axis:-



Consider two diametrically opposite elementary displacements located at A & B. Let point of observation ‘P’ be along ‘Z’ axis.

$$\boxed{\vec{E}_{at\ P} = \frac{\rho_L\ az}{2\varepsilon_o(a^2+z^2)^{3/2}} . \hat{z}}$$

Electric field due to an infinite charge sheet:



Consider two diametrically opposite elementary surface charges located at A & B. Let point of observation ‘P’ be along Z axis.

$$\boxed{\vec{E} = \frac{\rho_s}{2\varepsilon_o} . \hat{z}}$$

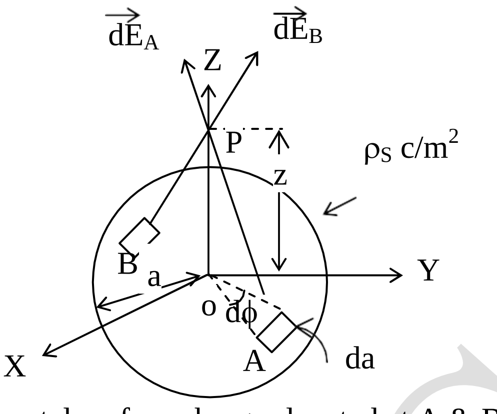
The electric field due to the surface charge sheet is independent of the distance of the point of observation (P) from the surface charge sheet. It has a constant magnitude equal to $\rho_S/2\varepsilon_0$ and has a direction normal to the surface charge sheet.

The field direction is away from the surface charge sheet towards the point of observation if it is a +ve charge sheet.

(Contd …17)

ECE-SAC

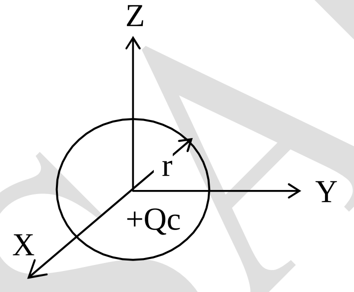
Electric field due to a circular disc along its axis:-



Consider two diametrically opposite elemental surface charges located at A & B. Let point of observation 'P' be along the z- axis.

$$\boxed{E_{at\ p} = \frac{\rho_s}{2\varepsilon_0}\left(1 - z / \sqrt{a^2 + z^2}\right) \hat{z}}$$

Gauss's Law:



Let us consider a point charge of '+Q'C at origin. Consider a closed surface.

The electric field at any point over the closed surface

$$\vec{E} = (Q / 4\pi\varepsilon_0 r^2) \,.\, \hat{r}$$

Differential area, $\vec{da} = r^2 \text{Sin}\theta d\theta d\phi \, \hat{r}$

$$\oint_s \vec{E} \,.\, \vec{da} = \frac{Q}{4\pi\varepsilon_0 \cancel{r^2}} \cancel{r^2} \int_0^{\pi} \text{Sin}\theta \, d\theta \int_0^{2\pi} d\phi$$

$$\oint_s \vec{E} \,.\, \vec{da} = (Q/\varepsilon_0)$$

Though the above result is deduced with respect to a spherical closed surface enclosed a point charge, it is a general result applicable for any closed surface enclosing any charge in any form.

$$\boxed{\oint_s \vec{E} \,.\, \vec{da} = (1/\varepsilon_0) \times Q_{enclosed} = (1/\varepsilon_0) \int_v \rho_v \, dv}$$

→ Gauss law in integral form (or) Maxwell's 1st equation

Using divergence theorem,

$$\int_v (\nabla \,.\, \vec{E}) \, dv = (1/\varepsilon_0) \int_v \rho_v \, dv$$

$$\therefore \quad \boxed{\nabla \,.\, \vec{E} = (\rho_v / \varepsilon_0)}$$ Ⓞ point form of Gauss law

(Contd …18)

ECE-SAEC

substitute D = εE

$$\oint_s \vec{D} \cdot \vec{da} = Q_{enclosed} = \int_v \rho_v \, dz$$

(or)

$$\boxed{\nabla \cdot \vec{D} = \rho_v} \longrightarrow \textbf{Maxwell's 1}^{st}\textbf{ Equation}$$

Statement:-

Surface integral of normal component of electric field Vector is equal to $(1/\varepsilon_o)$ times charge enclosed.

(or)

Surface integral of normal component of electric flux density vector is equal to the charge enclosed.

Gaussian Surface:

Gauss's law is very useful to find out electric field intensity. To find we construct an imaginary surface called "Gaussian Surface"

The electric field must be uniform at every point on this surface. It must be normal to the surface considered.

OBJECTIVES

One mark Questions

1) Inside a hollow conducting sphere (Gate – 96)
   a) electric field is zero
   b) electric field is a non-zero constant
   c) Electric field changes with the magnitude of the charge given to the conductor.
   d) Electric field changes with distance from the center of the sphere

2) A metal sphere with 1m radius and a surface charge density of 10 c/m^2 is enclosed in a curve of 10m side. The total outward electric displacement normal to the surface of the cube is (Gate – 96)
   a) 40π coulombs b) 10πcoulombs c) 5 coulombs d) none

3) If V,W,Q stands for Voltage, energy and charge, then V can be expressed as (Gate – 96)
   a) $V = \frac{dq}{dw}$ b) $V = \frac{dw}{dq}$ c) $dV = \frac{dw}{dq}$ d) $dV = \frac{dq}{dw}$

4) In the infinite plane, y=6m, there exists a uniform surface charge density of $(1/600\pi)\ \mu\ c/m^2$. The associated electric field strength is (Gate – 95)
   a) $30\ \hat{i}$ V/m b) $30\ \hat{j}$ V/m c) $30\ \hat{k}$v/m d) $60\ \hat{J}$ v/m

5) The electric field strength at a distance point, P due to a point charge, +q, located at the origin, is 100μV/m. If the point charge is now enclosed by a perfect conducting metal sheet sphere whose center is at the origin, then the electric field strength at the point , P outside the sphere bcomes
   a) zero b) 100 μ V/m c) –100 μV/m d) 50 μV/m

6) Copper behaves as a (Gate – 95)
   a) Conductor always
   b) Conductor or dielectric depends on the applied electric field strength
   c) Conductor or dielectric depends on the frequency
   d) Conductor or dielectric depends on the electric current density.

(Contd …19)

ECE-SAEC

7) Given the potential function in free space to be v(x) = $50x^2 + 50y^2 + 50z^2$ volts, the magnitude (in v/m) and the direction of electric field at point (1, -1,1), where the dimensions are in meters, are (Gate – 01)

a) 100; (i+j+k) b) 100/√3; (i-j+k)
c) 100/√3; [(-i +j-k)/√3] d) 100/√3; [(-i –j –k)/√3]

8) In a uniform electric field, field lines and equipotentials (Gate- 94)

a) are parallel to one another b) intersect at 45°
c) intersect at 30° d) are orthogonal

9) When a charge is given to a conductor (Gate –94)

a) It distributes uniforming all over the surface b) It distributes uniformly all over the volume
b) It distributes on the surface, inversely proportional to the radius of curvature
c) It stays where it was placed.

10) The mks unit of electric field E is (IETE)

a) Volt b) volt/second c) volt/metre d) ampere/metre

11) Unit of displacement density is

a) c/m b) c/m^2 c) Newton d) Maxwell's equation

12) Two infinite parallel metal plates are charged with equal surface charge density of the same polarity. The electric field in the gap b/w the plates is

a) The same as that produced by one plate b) Double of the field produced by one plate
b) Dependent on coordinates of the field point d) Zero

13) Three concentric spherical shells of Radii R1, R2,R3(R1<R2<R3) carry charges –1,-2,and 4 coulombs, respectively. The charge in coulombs on the inner and outersurfaces respectively, of the outermost shell is. (IES – 95)

a) 0 and 4 b) 3 and 1 c) –3 and 7 d) –2 and 6

14) A positive charge of 'Q' coulombs is located at point A(0,0,3) and a negative charge & magnitude Q coulomb is located at point B (0,0,-3). The electric field intensity at point c(4,0,0) is in the

a) negative X-direction b) negative Z-direction
c) positive X-direction d) positive Z-direction

15) The force between two point charges of 1nc each with a 1mm separation in air is (IES- 01)

a) 9×10^{-3} N b) 9×10^{-6}N c) 9×10^{-9}N d)9×10^{-12}N

16) Two charges of equal magnitudes are separated by some distance. If the charges are increased by 10%; to get the same force b/w them, their separation must be

a) increased by 21% b) increased by 10%
c) decreased by 10% d) non of the above is correct

Two mark Questions

Common data for Q. No. 17, 18 & 19

A small isolated conducting sphere of radius r_1 is charged with +Qc. Surrounding this sphere and concentric with it is a conduction spherical cell, which posses no net charge. The inner radius of the shell is r2, and outer radius r3. All non-conducting space is air.

17) The electric field distribution from 0 to r1 will be

a)zero b) same c)increases d)decreases

(Contd …20)

ECE-SAEC

18) The electric field from r1 to r2 will be

a) zero b) same c)decreases d) increases

19) The electric field from r2 to r3 will be

a) same b) zero c) decreasing d) increasing

Common data for Q. No. 20 & 21

Infinite surface charge sheets are placed along the Y-axis with surface charge density $+\rho_s$ c/m^2 and $-\rho_s$ c/m^2 respectively.

20) The electric field intensity between the sheets will be

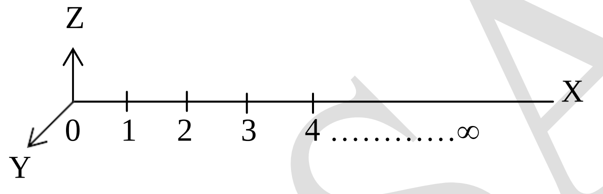
a) zero b) $\frac{\rho_s}{2\varepsilon_0}\,j$ c) $\frac{\rho_s}{\varepsilon_0}\,-j$ d) $\frac{\rho_s}{\varepsilon_0}\,j$

21) The electric field intensity outside the sheets will be

a) zero b) $\frac{\rho_s}{2\varepsilon_0}\,j$ c) $\frac{\rho_s}{\varepsilon_0}\,-j$ d) $\frac{\rho_s}{\varepsilon_0}\,j$

Common data for Q. No. 22 & 23

Infinite charges are placed along the X – axis at x = 1, 2, 3, ... ∞



22) An infinite number of charges, each equal to ‘Q’ c, the electric field at the point x = 0 due to these charges will be

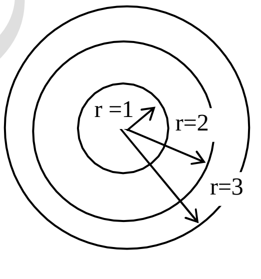
a) Q b) 2Q / 3 c) 4Q/3 d) 4Q/5

23) The electric field at x = 0, when the alternate charges are of opposite in nature, will be

a) 4Q/3 b) 4Q/5 c) 1.5Q d) 3Q

Common data for Linked answer

The spherical surfaces r = 1, 2 & 3 carry surface charge densities of 20 nc/m^2, -9 nc/m^2 and $2nc/m^2$ respectively.



24) How much electric flux leaves the surface at r = 5 ?

a) $2\pi \times 10^{-3}$ b) 8π c) $3\pi \times 10^{-9}$ d) $8\pi \times 10^{-9}$

25) Find electric flux density at P(1, -1, 2)

a) $8.83 \times 10^{-9}\,\hat{r}$ b) $3.3 \times 10^{-10}\,\hat{r}$ c) $3.8 \times 10^{-3}\,\hat{r}$ d) $40 \times 10^{-9}\,\hat{r}$

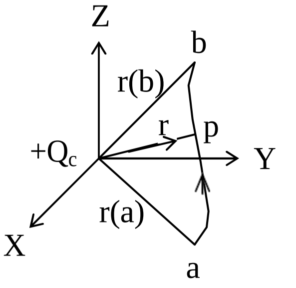
Key:

1. a	2. a	3. b	4. c	5. c	6.a	7. c	8. d	9. a	10. c	11. b	12. d	13. b
14. b	15.a	16. b	17. a	18. c	19. b	20. d	21. a	22. c	23. b	24. d	25. b	

(Contd ...21)

ECE-SAEC

Electric Potential:



Consider $+Q_C$ of charge at origin.
Let the point of observation is at a distance 'r' from the origin on the open path ab.
We know,

$$\vec{E}_{at\ p} = (Q / 4\pi\varepsilon_0 r^2) \cdot \hat{r}$$

Displacement vector $\vec{dl} = (dr)\,\hat{r} + (rd\theta)\,\hat{\theta} + (r\sin\theta d\phi)\,\hat{\phi}$

$$\therefore \quad \boxed{\int_a^b \vec{E} \cdot \vec{dl} = (Q / 4\pi\varepsilon_0)[1/r(a) - 1/r(b)]}$$

The integral $\vec{E} \cdot \vec{dl}$ is independent of the open path and depends only on the starting and ending point. Now let the starting point be replaced by a reference point (θ) and the ending point be replaced by the point of observation (p).

The quantity $\int_\theta^p \vec{E} \cdot \vec{dl}$ attached with a 'negative' sign is known as electric potential at the point of observation p.

$$\therefore \quad \boxed{V(p) = -\int_\theta^p \vec{E} \cdot \vec{dl}}$$

Note: For finite charge distribution, 'infinity' is recommended as the reference point and for "infinite charge distribution" other than infinity can be assumed as the reference point.

Potential difference between two points:

$$\boxed{V(A) - V(B) = -\int_B^A \vec{E} \cdot \vec{dl} = \int_A^B \vec{E} \cdot \vec{dl}}$$

Relation between electric potential and Electric field (V & E):

We know that,

$$V(A) - V(B) = \int_A^B \vec{E} \cdot \vec{dl} \quad \text{---} \quad (1)$$

The fundamental theorem of gradient,

$$V(B) - V(A) = \int_A^B (\nabla V) \cdot \vec{dl}$$

$$V(A) - V(B) = -\int_A^B (\nabla V) \cdot \vec{dl} \quad \text{---} \quad (2)$$

(Contd …22)

ECE-SAEC

Compare (1) & (2)

$$\boxed{\vec{E} = -\nabla V}$$

i) Taking 'curl' on both sides

$$\nabla \times E = \nabla \times (-\nabla V)$$

$$\therefore \quad \boxed{\nabla \times \vec{E} = 0} \longrightarrow$$ **Maxwell's 3rd Equation**

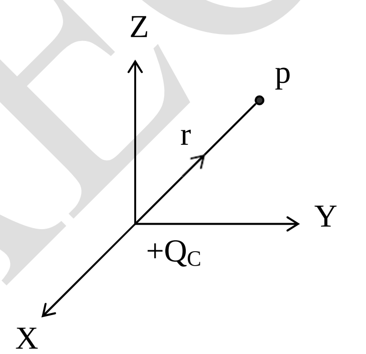
ii) Taking 'divergence' on both sides

$$\nabla \cdot E = \nabla \cdot (-\nabla V)$$
$$= \nabla^2 V \neq 0$$

$$\therefore \quad \boxed{\nabla \cdot \vec{E} \neq 0}$$

$\therefore$ Therefore, an electrostatic field is irrotational (or) conservative but not solenoidal.

Electric potential due to a point charge (Absolute potential):



We know,

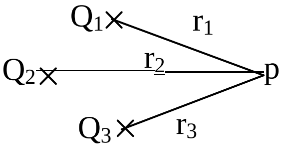
$$V(p) = -\int_{\theta}^{p} \vec{E} \cdot d\vec{l}$$

Due to finite charge, replace reference point θ with infinity (∞).

$$V(p) = -\int_{\infty}^{r} (Q / 4\pi\varepsilon_0 r^2) \cdot dr$$

$$\therefore \quad \boxed{V(p) = Q / 4\pi\varepsilon_0 r}$$

Electric potential due to a discrete charges:



$$V(p) = V(Q_1) + V(Q_2) + V(Q_3) + \ldots\ldots$$

$$= Q / 4\pi\varepsilon_0 r_1 + Q / 4\pi\varepsilon_0 r_2 + \ldots\ldots.$$

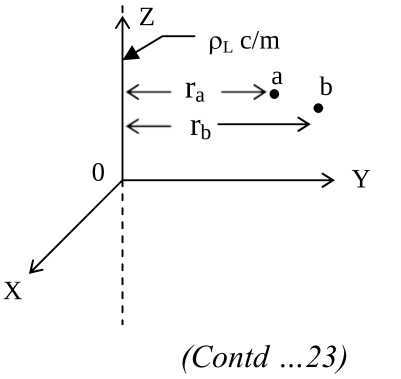
Electric Potential due to a continuous charge distribution:

$V(p) = \int (\rho_L\, dl) / 4\pi\varepsilon_0 r$ ◎ for line charge distribution

$= \oint_s (\rho_s\, da) / 4\pi\varepsilon_0 r$ ◎ for surface charge distribution

$= \int_v (\rho_v\, dv) / 4\pi\varepsilon_0 r$ ◎ for volume charge distribution

Electric potential due to an infinite line charge distribution:



Consider an infinite line charge placed along the Z – axis.

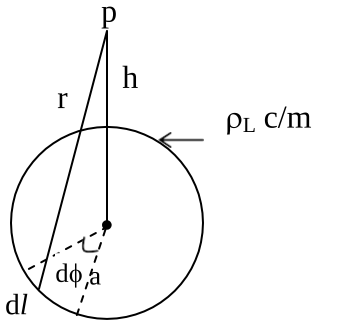
$\therefore$ Electric potential difference,

$$\therefore \quad \boxed{V = (\rho_L / 2\pi\varepsilon_0)\, ln\, (r_b/r_a)}$$

(Contd …23)

ECE-SAEC

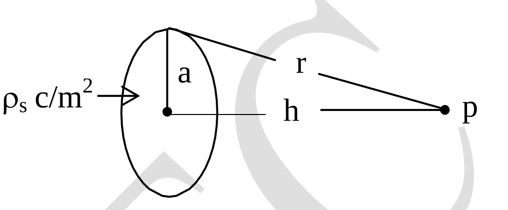
Electric potential due to a charged ring:



$\therefore \quad \boxed{V = (\rho_L a) / (2\varepsilon_0 r)}$

where $\boxed{r = \sqrt{a^2 + h^2}}$

Potential due to a charged disc:



Electric potential at p,

$\therefore \quad \boxed{V = (\rho_s / 2\varepsilon_0)[\sqrt{a^2 + h^2} - h]}$

i) Potential at the centre of the disc,

substitute h = 0

$\therefore \quad \boxed{V = (\rho_s a / 2\varepsilon_0)}$

Poisson's equation and Laplace's equation:

From the differential form of Gauss law,

$\nabla . \vec{E} = \rho / \varepsilon_0$ --- (1)

But,

$\vec{E} = -\nabla V$ --- (2)

Substitute,

$\nabla.(-\nabla V) = \rho / \varepsilon_0$

$\therefore \quad \boxed{\nabla^2 V = -\rho / \varepsilon_0} \Rightarrow$ Poisson's equation

For a charge free region i.e., $\rho = 0$

$\boxed{\nabla^2 V = 0} \Rightarrow$ Laplace's equation

Both these equations are effectively used to determine the potential and electric field distribution without knowledge of source charge distribution.

Solution to Laplace's equation in Cartesian Co – Ordinates:

Laplace equation, $\nabla^2 V = 0$

$\Rightarrow (\partial^2 V / \partial x^2) + (\partial^2 V / \partial y^2) + \partial^2 V / \partial Z^2) = 0$

Case1: 'V' is a function of only 'x'

$\therefore \quad \boxed{V = Ax + B}$

Case2: 'V' is a function of only 'y'

$\therefore \quad \boxed{V = Ay + B}$

Case3: 'V' is a function of only 'z'

$\therefore \quad \boxed{V = Az + B}$

(Contd …24)

ECE-SAEC

Solution of Laplace equation in spherical co – ordinates:

$$\nabla^2 V = 0$$

$$\Rightarrow 1/r^2 \mathrm{Sin}\theta \, [\, \partial/\partial r(r^2 \sin\theta \, \partial V/\partial r) + \partial/\partial\theta(\mathrm{Sin}\theta \, \partial V/\partial\theta) + \partial/\partial\phi[(1/\mathrm{Sin}\theta)\, \partial V/\partial\phi)] = 0$$

Case1: ‘V’ is a function of ‘r’ only

$$\therefore \qquad \boxed{V = -A / r + B}$$

Case2: ‘V’ is a function of ‘θ’ only

$$\therefore \qquad \boxed{V = A \, ln \tan(\theta/2) + B}$$

Case3: ‘V’ is a function of ‘ϕ’ only

$$\therefore \qquad \boxed{V = A\phi + B}$$

Solution of Laplace equation in Cylindrical Co – ordinates:

$$\nabla^2 V = 0$$

$$\Rightarrow 1/r[1/\partial r(r \, \partial V/\partial r) + \partial/\partial\phi(1/r \, . \, \partial V/\partial\phi) + \partial/\partial z(r \, \partial V/\partial z)] = 0$$

Case1: ‘V’ is a function of ‘r’ only

$$\therefore \qquad \boxed{V = A \, ln \, r + B}$$

Case2: ‘V’ is a function of ‘ϕ’ only

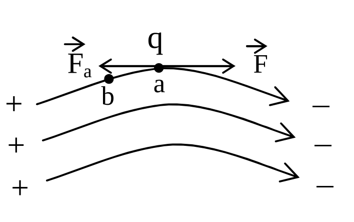
$$\therefore \qquad \boxed{V = A\phi + B}$$

Case3: ‘V’ is a function of ‘z’ only

$$\therefore \qquad \boxed{V = Az + B}$$

Note: Here A and B are arbitrary constants, whose values are determined by using appropriate boundary conditions.

Work Done:



A charge ‘q’ kept in the electric field experiences a force in the direction of electric field. F is the force experienced by the charge ‘q’. F_a is the force applied in opposite direction. If the magnitude of F_a is equal to F, the charge remains in equilibrium. If F_a is slightly greater than F, the charge can be moved from point a to point b. The small work done to move the charge ‘q’ by a distance ‘d*l*’ is $F_a.dl$. Total work done in moving the charge from a to b can be obtained.

(Contd …25)

ECE-SAEC

$$\text{Work done} = \int \vec{F}_a \cdot d\vec{l} \qquad \text{Where } \vec{F}_a = -F$$

$$= \int_a^b \vec{F}_a \cdot d\vec{l} = -q \int_a^b \vec{E} \cdot d\vec{l} \qquad [\because \vec{F} = E\vec{q}]$$

$\therefore$

$$\boxed{\text{Work done} = -q \int_a^b \vec{E} \cdot d\vec{l}}$$

Energy: If point ‘a’ is replaced by the reference point ‘θ’ and point ‘b’ is replaced by point of observation (p), then

$$\boxed{W = -q \int_\theta^p \vec{E} \cdot d\vec{l} = q\,V(p)}$$

$$\text{Where } V(p) = -\int_\theta^p \vec{E} \cdot d\vec{l}$$

The above expression represents the energy because this amount of work done is stored in the form of electrostatic energy.

Energy stored in a system of ‘n’ point charges:

Consider a system having ‘n’ number of point charges.

Energy stored in this system = ½ ($V_1Q_1 + V_2Q_2 + \ldots\ldots + V_nQ_n$)

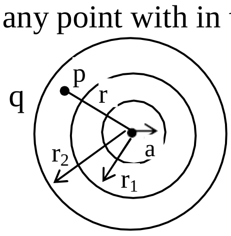
In compact form

$$\boxed{W = 1/2 \sum_{i=1}^{n} Q_i V_i}$$

OBJECTIVES

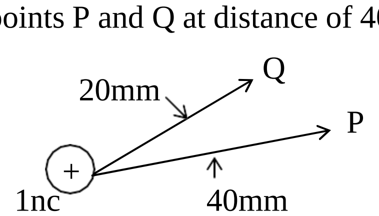
One Mark Questions

1. A spherical conductor of radius ‘a’ with, charge ‘q’ is placed concentrically inside an uncharged and unearthed spherical conducting shell of inner and outer radii r_1 and r_2 respectively. Taking potential to be zero at infinity, the potential any point with in the shell ($r_1 < r < r_2$) will be ***(GATE’95)***
   a) $q / 4\pi\varepsilon_0 r$
   b) $q / 4\pi\varepsilon_0 a$
   c) $q / 4\pi\varepsilon_0 r_2$
   d) $q / 4\pi\varepsilon_0 r_1$



2. Which of the following equation(s) is/are correct?
   a) $\overline{J} = \sigma\overline{E}$ b) $\nabla\overline{V} = \overline{E}$ c) $\overline{D} = \in\overline{E}$ d) all the above

3. A point charge of +1nc is placed in a space with a permittivity of 8.85×10^{-12} F/m as shown. The potential difference V_{PQ} between two points P and Q at distance of 40mm and 20mm respectively from the point charge is ***(GATE’03)***
   a) 0.22 KV
   b) – 225 V
   c) – 2.24 KV
   d) 15 V



(Contd …26)

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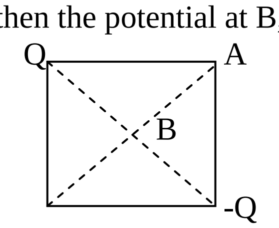
4. One volt equals *(BEL'95)*
a) one Joule b) One Joule / Coulomb c) One Coulomb / Joule d) None

5. Equation $\nabla^2 V = -\rho/\in$ is called the *(IIT)*
a) Poisson's equation b) Laplace equation c) Continuity equation d) None

6. Two point charges Q and –Q are located on two opposite corners of a square as shown. If the potential at the corner A is taken as 1V, then the potential at B, the centre of the square will be *(IES'93)*
a) zero
b) $1/\sqrt{2}$ V
c) 1 V
d) $\sqrt{2}$ V



7. The potential inside a charged hollow sphere is
a) zero b) same as that on the surface c) less than that on the surface d) none

8. Two spheres of radii 'r_1' and 'r_2' are connected by a conducting wire. Each of the spheres has been given a charge Q. Now,
a) larger sphere will have greater potential b) larger sphere will have smaller potential
c) both the spheres will have same potential d) smaller sphere will have zero potential

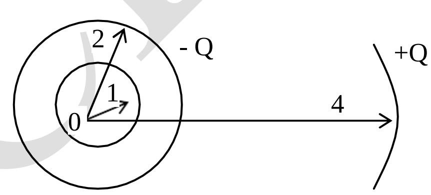
9. Potential of a sphere is given as
a) $Q / 4\pi\varepsilon_0 r$ b) $Q / \pi\varepsilon_0 r$ c) $Q / 4\pi\varepsilon_0 r^2$ d) $Q^2 / 4\pi\varepsilon_0 r^2$

10. A sphere of radii 1m can attain a maximum potential of
a) 3×10^6 V b) 30 KV c) 1000 V d) 3 KV

11. Joule / Coulomb is the unit of
a) electric field intensity b) potential c) charge d) None

Two Mark Questions

12.



An infinite number of concentric rings carry a charge Q each alternately positive and negative. Their radii are 1,2,4,8,…. metres in geometric progression as shown. The potential at the centre of the rings will be *(IES'92)*

a) zero b) $Q / 12\pi\varepsilon_0$ c) $Q / 8\pi\varepsilon_0$ d) $Q / 6\pi\varepsilon_0$

13. Find the work involved in moving a charge of 1C from (6,8,-10) to (3,4,-5) along a straight line in the field $E = -x\hat{i} + y\hat{j} - z\hat{k}$.
a) 24.5 Joules b) 25.5 Joules c) 19 Joules d) zero

14. Find the work done in moving a point charge 3 μc from (2,π, 0) to (4, π,0) in the field $E = 10^5/r\,\hat{r} + 10^5 z\,\hat{z}$.
a) 0.207 Joules b) 1.27 Joules c) 0.8 Joules d) zero

15. Five equal point charges of zone are located x = 2,3,4,5 and 6 m. Find the potential at the origin.
a) 180 V b) 183 V c) 210 V d) 261 V

(Contd …27)

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16. A line charge of $10^{-9}/2$ c/m lies on the Z – axis. Find r_{ab} if 'a' is at (2,0,0) and b is at (4,0,0)
a) 2V b) 4.24 V c) 6.24 V d) 8.24 V

17. A point charge of 0.4 nc is located at (2,3,3) in Cartesian system. Find r_{ab} if A is (2,2,3) and B is (-2,3,3).
a) 2.7 V b) 3.6 V c) 4.7 V d) 8.1 V

18. Determine the potential at (0,0,5) m caused by a total charge 10^{-8}c distributed uniformly along a disc of radius 5m lying in the Z = 0 plane and centered at the origin.
a) 12.2 b) 17 V c) 14.8 V d) 13.2 V

19. 3 point charges of 1C, 2C and 3C are located at the corner of an equilateral triangle of 1m side each. Find the energy stored in the system.
a) $9 / 4\pi\in_0$ Joules b) $4\pi\in_0 / 3$ Joules c) $11 / 4\pi\in_0$ Joules d) 30×10^9 Joules

20. If the potential is given by $V = 5r^2$ where 'r' is distance from origin. How much charge is located with in a sphere of 1m radius centered at the origin.
a) $90 \in_0$ b) $-30\in_0$ c) $30 \in_0$ d) $-30 / \in_0$

Common data question

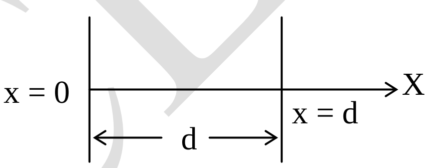
A spherical shell of radius 'a' contains a total charge of Q_0 uniformly distributed over its surface.

21. Find the potential inside the spherical shell
a) $Q_0^2 / 4\pi$ b) $Q_0 / 4\pi^2\varepsilon_0 a$ c) $Q_0 / 4\pi\varepsilon_0 a$ d) zero

22. Find the potential outside the spherical shell
a) $Q_0^2 / 4\pi$ b) $Q_0 / 4\pi\varepsilon_0 a$ c) zero d) $Q_0 / 4\pi\varepsilon_0 r$

Linked Question

Two parallel infinite conducting plates separated by a distance 'd' along the X – axis have a potential V_0 and zero respectively as shown.



23. Find the expression for voltage distribution
a) $V = V_0(1 + d/x)$ b) $V = V_0(1 - x/d)$ c) $V = V_0(1 - d/x)$ d) 0

24. Find the electric field intensity
a) $(V_0 / x)\hat{i}$ b) $V_0\hat{i}$ c) $(V_0 / d) . \hat{i}$ d) $(x / V_0) . \hat{i}$

Key:

1.a	2.d	3.b	4.b	5.a	6.c	7.b	8.c	9.a	10.a	11.b	12.d	13.b
14.a	15.d	16.c	17.a	18.c	19.c	20.b	21.c	22.d	23.b	24.c		

(Contd …28)

ECE-SAEC

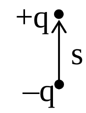
Polar and Non – Polar Dielectrics:

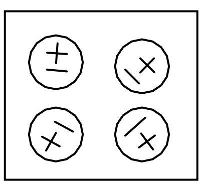
Dielectric is nothing but an insulator. It is capable of storing energy for a short duration. Dielectrics are classified as polar and non – polar type.

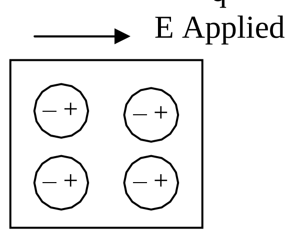
Electric Dipole: Two equal and opposite charges separated by a small distance is called a dipole.

Dipole Moment: Dipole moment is a product of charge and distance between charges.

Dipole moment $\boxed{\vec{p} = q\vec{s}}$







Fig(1)
Polar Dielectric without applied electric field

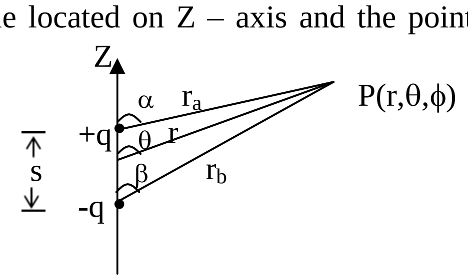
fig(2)
Polar Dielectric with applied Electric field

Polar Dielectrics: The charges in the molecules of polar type have permanent displacement from each other. The molecules have permanent dipole moment. They are randomly oriented as shown in fig(1). Net dipole moment zero until an electric field is applied.

When an electric field is applied, the dipoles orient in a particular direction such that the induced electric field is in a direction opposite to the applied electric field. This can be seen in fig(2).

Non – Polar Dielectrics: In non – polar dielectrics, the centres of positive and negative charges coincide each other. When non – polar dielectric is kept in the electric field, a small displacement takes place between the charges.

Potential due to a dipole: Let us consider a physical dipole located on Z – axis and the point of observation P(r, θ, ϕ).



It is required to determine the potential at ‘p’ which is at a distance ‘r’ m from the midpoint of the dipole. It is easy to handle this problem using spherical co – ordinates.

Potential at ‘p’ is the sum of potential values due to positive and negative charges.

Therefore, the potential at ‘p’ due to the physical dipole is given by

$$V(p) = V_1 + V_2$$
$$= q / (4\pi\varepsilon_0 r_a) + (-q) / (4\pi\varepsilon_0 r_b)$$
$$\therefore \quad V(p) = q / (4\pi\varepsilon_0)[1/r_a - 1/r_b]$$

$$\therefore \quad V(p) = q/(4\pi\varepsilon_0)[(r_b – r_a) / r_a r_b]$$

(Contd …29)

ECE-SAEC

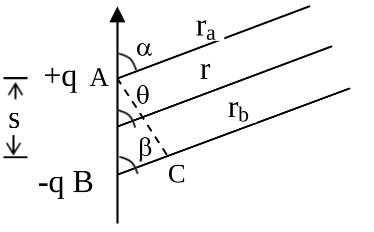
Case: When the point of observation is at a very large distance $\alpha = \beta = \theta$ and $r_a = r_b = r$

$r_b - r_a = BC$

$\therefore \quad V(p) = q / (4\pi\varepsilon_0)\,[BC / r_a r_b]$

$= q / (4\pi\varepsilon_0)\,[S\cos\theta / r^2] \qquad [\because BC = S\cos\theta]$

$$\boxed{V(p) = (p\cos\theta) / (4\pi\varepsilon_0 r^2)} \qquad [\because \vec{p} = q\vec{S}]$$



$$\boxed{V(p)\ \alpha\ 1/r^2}$$

Electric field intensity due to a Dipole:

We know $\quad \vec{E} = -\nabla V$

$\therefore \quad \boxed{\vec{E} = p / (4\pi\varepsilon_0 r^3)[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}]}$ in spherical system

$\therefore \quad \boxed{\vec{E} \propto (1/r^3)}$

Observations:

i) Potential due to an electric dipole $V(p) \propto 1 / r^2$

ii) Electric field intensity due to an electric dipole $E \propto 1/r^3$

Polarization ($\vec{P}$)

Some materials already contain the internal electric dipoles. When such materials are subjected to an electric field these internal electric dipoles align themselves along the direction of applied electric field.

Many materials do not contain any internal electric dipoles. When such materials are subjected to an electric field, internal electric dipoles are generated and align themselves along the direction of applied electric field.

Qualitatively defined as production and / or alignment of internal electric dipoles.

Quantitatively defined as effective dipole moment per unit volume.

$\therefore \quad \boxed{\vec{P} = \vec{p} / dv}$

units for polarization is coulomb / m^2.

Susceptibility(χ):

Susceptibility is one less than relative permittivity.

$\chi = \varepsilon_r - 1$

Displacement density is directly proportional to electric field intensity.

$D \propto E$

$D = \varepsilon_0\,\varepsilon_r\,E \qquad \text{----} \qquad (1)$

When a dielectric is kept in the electric field, a net dipole moment exist since the dipoles align in one particular direction in the case of polar dielectrics. Polarization density (p) is directly proportional to the applied electric field.

$P = \chi\,\varepsilon_0\,E \qquad \text{----} \qquad (2)$

From (1) & (2)

$\dfrac{P}{D} = \dfrac{\chi\varepsilon_0 E}{\varepsilon_0\varepsilon_r E}$

$= \chi / \varepsilon_r$

$= (\varepsilon_r - 1) / \varepsilon_r \qquad [\because \chi = \varepsilon_r - 1]$

$\therefore \quad \boxed{P = [(\varepsilon_r - 1) / \varepsilon_r]\,.\,D}$

(Contd …30)

ECE-SAEC

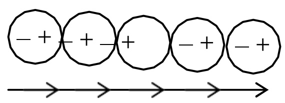
Gauss's Law for Dielectrics:

We know that differential form of Gauss law in free space.

$$\boxed{\nabla \cdot \vec{E} = \rho_f / \varepsilon_0}$$

Where ρ_f ⓪ free volume charge density.

Consider a row of dipoles as shown.



The positive charge is nullified by the negative charge near by (or) the head and tail gets cancelled throughout except at the beginning and end. In other words, a negative charge and a positive charge can be seen at the boundaries. This charge is called 'Bound charges'.

Gauss's law is modified as follows.

$$\boxed{\nabla \cdot \vec{E} = (\rho_f + \rho_b) / \varepsilon_0}$$

Where ρ_b ⓪ bounded volume charge density

Statement: Surface integral of normal component of electric field is equal to $1/\in_0$ times the sum of free charge and bound charge.

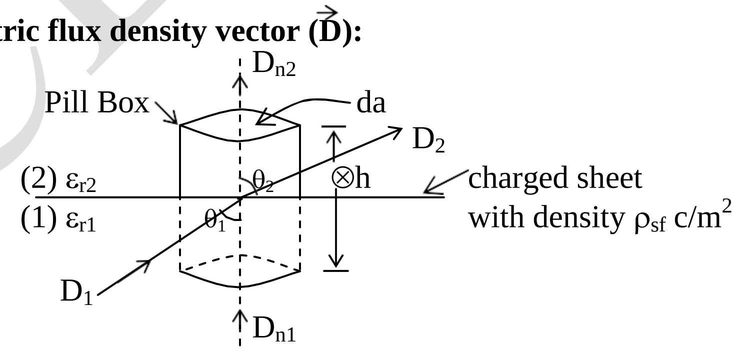
$$\boxed{\oiint_S \vec{E} \cdot \vec{da} = 1/\varepsilon_0 \, (Q_f + Q_b)}$$

Point form of Gauss's law is, $\boxed{\nabla \cdot D = \rho_f + \rho_b}$

Dielectric Boundary Conditions:

When flux lines flow through a single medium, they are continuous. When they flow through a boundary formed by two different types of dielectrics, they get refracted. This can be studied by using boundary conditions. Surface of glass board is glass air boundary. Surface of porcelain insulator is a porcelain air boundary.

Boundary condition for Electric flux density vector ($\vec{D}$):



Consider a boundary formed by two dielectrics as shown in the figure. An infinite charged sheet with charge density ρ_s c/m^2 is placed at the boundary. The dielectric constants of the media 1 and 2 are $\in r_1$ and $\in r_2$ respectively. θ_1 is the angle of incidence. θ_2 is the angle of emergence. D_{n1} and D_{n2} are the normal components of flux density vectors.

Consider a pill box at the boundary such that it encloses both the media. Apply Gauss's law to the pill box under limiting condition ⊗h⓪0.

$$\oiint_S \vec{D} \cdot \vec{da} = Q_f \text{ enclosed}$$

$$D_{n2} \int da - D_{n1} \int da = \rho_{sf} \times A$$

$$D_{n2}A - D_{n1}A = \rho_{sf} \, A$$

(Contd …31)

ECE-SAEC

$$\therefore \quad \boxed{D_{n2} - D_{n1} = \rho_{sf}}$$

Statement: Normal component of flux density vector is discontinuous by an amount equal to the charge density of the sheet.

If charged sheet is not present at the boundary, $\rho_{sf} = 0$.

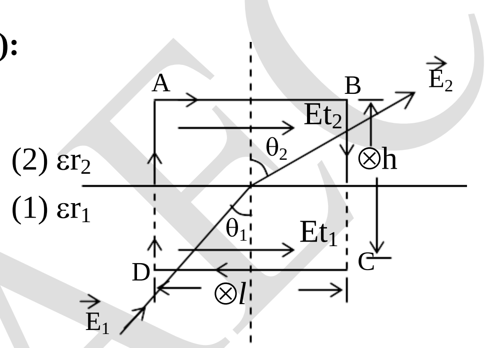
$$\therefore \quad \boxed{D_{n1} = D_{n2}}$$

Statement: Normal components of flux density vectors are equal. They are continuous at the boundary provided there is no charged sheet at the boundary.

Boundary condition for Electric field intensity vector($\vec{E}$):

Second boundary condition deals with tangential component of electric field. E_1 and E_2 are the electric field intensities in the media 1 and 2 respectively. E_{t1} and E_{t2} are the tangential components of the electric field in media 1 and 2 respectively.

Consider the rectangular path ABCDA at the boundary such that it encloses both the media.



We know that static electric field is a conservative field.

$$\oint \vec{E} \cdot \vec{dl} = 0$$

Apply this equation to the contour ABCDA under limiting condition ⊗h ◎ 0.

$$\int_{AB} \vec{E} \cdot \vec{dl} + \int_{BC} \vec{E} \cdot \vec{dl} + \int_{CD} \vec{E} \cdot \vec{dl} + \int_{DA} \vec{E} \cdot \vec{dl} = 0.$$

As ⊗h◎0, second and fourth terms tends to zero.

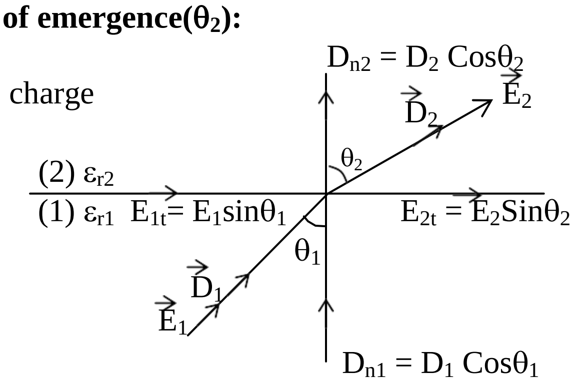
$$Et_2 \int dl - Et_1 \int dl = 0$$

$$Et_2 \otimes l - Et_1 \otimes l = 0$$

$$\therefore \quad \boxed{Et_1 = Et_2}$$

Statement: Tangential components of electric field intensity vector are equal and they are continuous at the interface.

Relation between angle of incidence (θ_1) and angle of emergence(θ_2):



Assume interface does not contain any surface charge

Apply boundary condition for D,

$$Dn_2 = Dn_1 \qquad [\because \rho_{sf} = 0]$$

$$D_2\cos\theta_2 = D_1 \cos\theta_1$$

$$\varepsilon_2 E_2 \cos\theta_2 = \varepsilon_1 E_1 \cos\theta_1 \quad ---- \quad (1)$$

Apply boundary condition for E,

$$E_{2t} = E_{1t}$$

$$E_2 \sin\theta_2 = E_1 \sin\theta_1 \quad ---- \quad (2)$$

(2) ÷ (1)

$$\frac{E_2 \sin\theta_2}{\varepsilon_2 E_2 \cos\theta_2} = \frac{E_1 \sin\theta_1}{\varepsilon_1 E_1 \cos\theta_1}$$

$$\therefore \quad \boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}}$$

If ε_{r1}, ε_{r2} and angle of incidence are given, angle of emergence can be calculated using the above equation.

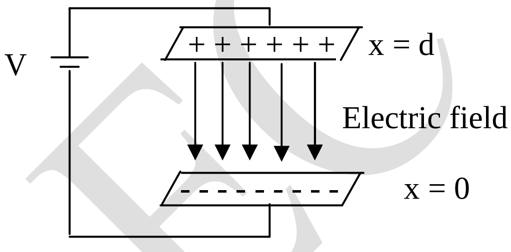
(Contd …32)

ECE-SAEC

Capacitor is formed using two conducting media with an insulator in between them.

Capacitance is the property of a dielectric to store electrical energy. An electric field is present between the plates since a voltage is applied between them. The dielectric is subjected to electric stress and strain. Therefore some energy can be stored in the dielectric. Capacitance is similar to inertia. The speed of a vehicle cannot change suddenly due to inertia. Similarly voltage across capacitor cannot change suddenly.

Capacitance of a parallel plate capacitor:



We know that, $c = Q/v$

$$= \frac{\left|\int \rho_s da\right|}{\left|-\int \vec{E}.\vec{dl}\right|}$$

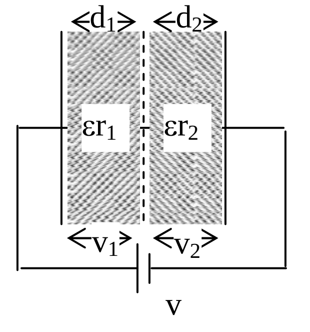
$$= \frac{\rho_s \text{ (Area)}}{\int \frac{\rho_s}{\varepsilon} \hat{i} . dx \hat{i}}$$

$$= \frac{\rho_s (A)}{\frac{\rho_s}{\varepsilon} \int_0^d dx}$$

$$= \frac{\rho_s (A)}{\frac{\rho_s}{\varepsilon} x d}$$

$$\boxed{\therefore c = \frac{\varepsilon A}{d}}$$ where A = cross section area of plate

Capacitance of parallel plate capacitor with two media



$$V = V_1 + V_2$$
$$= E_1 d_1 + E_2 d_2$$
$$= (D/\varepsilon_1)\, d_1 + (D/\varepsilon_2)\, d_2$$

$$= (Q/A\varepsilon_1)\, d_1 + (Q/A\varepsilon_2) d_2$$

$$= Q/A \left(\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} \right)$$

$$V = \frac{CV}{A} \left(\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} \right)$$

$$C = \frac{A}{\frac{d_1}{\varepsilon_0 \varepsilon_{r1}} + \frac{d_2}{\varepsilon_0 \varepsilon_{r2}}}$$

(Contd …33)

ECE-SALC

$$C = \frac{\varepsilon_0 A}{\left(\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}\right)}$$

Note: If n number of dielectrics are present, the equation can be written as

$$\therefore \boxed{C = \frac{\varepsilon_0 A}{\sum_{i=1}^{n} \frac{d_i}{\varepsilon_i}}}$$

Capacitance of Spherical capacitor:

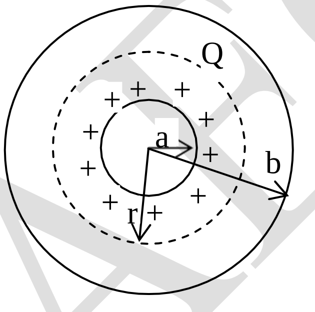
Consider a Gaussian sphere.

Apply Gauss's law

$D.da = Q_{enclosed}$

$D.\ 4\pi r^2 = Q$

$\therefore \vec{E} = Q/\ 4\pi\varepsilon_o r^2\ \hat{r}$



electric field exists only in the direction of r

we know, $v = -\int_b^a \vec{E}.\vec{dl} = -\int_b^a Q/4\pi\varepsilon r^2\ .\ dr$

$= Q/4\pi\varepsilon\ [1/a - 1/b]$

$$Q = \frac{4\pi\varepsilon\ v\ ab}{b - a}$$

substitute Q = CV

$$CV = \frac{4\pi\varepsilon\ v\ ab}{b - a}$$

$$\boxed{C = \frac{4\pi\varepsilon\ ab}{b - a}}$$

Capacitance of cylindrical capacitor (or) cable:

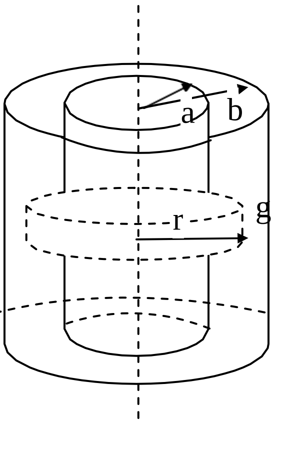
Consider a Gaussian cylinder (G).

Apply Gauss's law

$D.da = Q_{enclosed}$

$D.2\pi rl = Q$

$\vec{E} = \frac{Q}{2\pi\varepsilon r}\ r \qquad [\because l = 1\text{mt}]$



we know that,

$V = -\int_b^a \vec{E}.dr$

ECE-SAEC

(Contd …34)

$$= -\int_b^a \frac{Q}{2\pi\varepsilon r} dr$$

$$= \frac{Q}{2\pi\varepsilon} [\ln r]_a^b$$

$$= \frac{Q}{2\pi\varepsilon} [\ln b - \ln a]$$

$$V = \frac{Q}{2\pi\varepsilon} \ln (b/a)$$

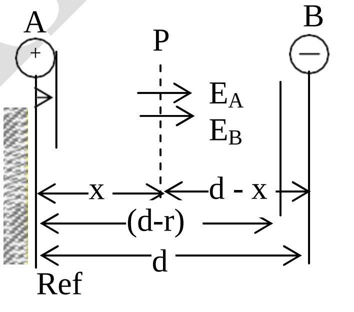
Substitute Q = CV

$$\boxed{C = \frac{2\pi\varepsilon}{\ln(b/a)}} \quad \text{Farad/m}$$

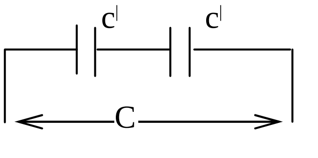
Note: To calculate the total capacitance, multiply with total length

Capacitance of a 2-wire transmission line:

Single phase transmission line is shown. Conductors A and B are uniformly charged with $+\rho_L$ c/m and $-\rho_L$ c/m respectively. Radius of each conductor is 'r' and spacing is 'd' meters. Consider a point 'P' at a distance 'x' meters from the reference. The distance between the wire B and the point P is (d-x). E_A and E_B are the electric field intensities due to wires A and B respectively. Direction of electric field is away from the positive charge or towards the negative charge.



$$\boxed{C = \frac{\pi\varepsilon}{\ln((d-r)/r)}} \quad \text{F/m}$$



Let C^1 be the capacitance per conductor.

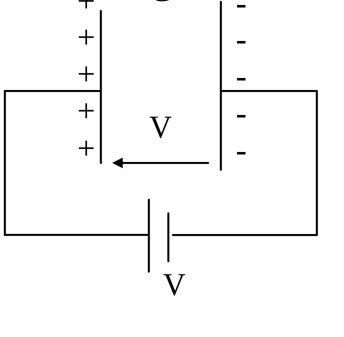
$$C = \frac{c^l \times c^l}{c^l + c^l} = \frac{c^l}{2}$$

$$c^l = 2c$$

$$\boxed{c^l = \frac{2\pi\varepsilon}{\ln((d-r)/r)}} \quad \text{F/m/conductor}$$

Energy stored in capacitor:

we know that energy stored in 'n' point system,



$$W = 1/2 \sum_{i=1}^{n} q\, v(p_i)$$

$$= 1/2 \int_s (\rho_s da)\, v$$

$$= 1/2\ \rho_s\, v \int_s da$$

$$= 1/2 \times \frac{Q}{\cancel{a}} \times V \times \cancel{a}$$

$$= 1/2\, QV \qquad [\because Q = CV]$$

(Contd …35)

$$\boxed{W = 1/2\ cv^2}\ \text{Joules}$$

$$\text{Energy density} = \frac{\text{Energy}}{\text{Volume}}$$

$$= \frac{1/2\ cv^2}{A \times d}$$

$$= 1/2 \times \frac{1}{\cancel{A}xd} \times \frac{\varepsilon A}{d} \cancel{x} v^2$$

$$= (1/2)\,\varepsilon\,(v/d)^2 \qquad [\because v/d = E]$$

$$= 1/2\ \varepsilon E^2$$

$$= 1/2\ (\varepsilon E)\ E$$

$$= 1/2\ DE$$

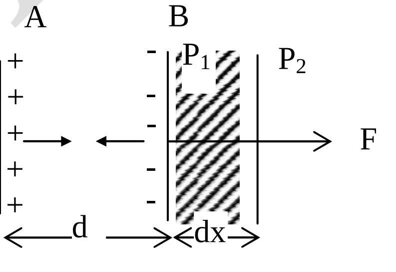
D and E can be written as D.E since D & E are in same direction

$\therefore$ Energy density = 1/2 D.E

Energy = 1/2 D.E x volume

$$\boxed{W = 1/2 \int_v \vec{D}.\vec{E}\ dv}$$

Force of Attraction between plates:



Between the oppositely charged plates there is a force of attraction. F is an externally applied force to move the plate B from p_1 to p_2. The work done is stored in the form of energy in the additional volume Adx.

Work done = Additional energy

F dx = (Energy density) volume

$F\ \cancel{dx} = (\ 1/2\ \varepsilon E^2)\ A\cancel{dx}$

$F = 1/2\ \varepsilon E^2 A$ Newtons

$F/A = 1/2\ \varepsilon E^2\ N/m^2$

$$\therefore \quad \boxed{\text{Force /unit area} = 1/2\ \varepsilon E^2}$$

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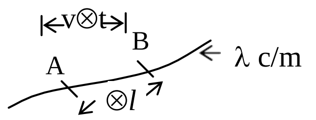
(Contd ...36)

Classification of currents:

For theoretical convenience currents can be classified into 3 types

a) Line currents b) Surface currents c) Volume currents

Line Currents



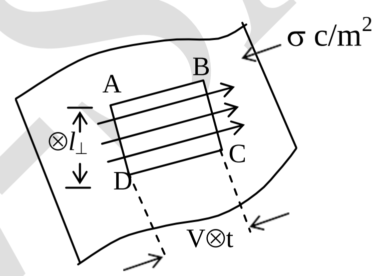
Motion of electric charges along a line represents a line current. Every line current is associated with a mobile line charge density λ c/m. An elementary segment $\otimes l = V \otimes t$ along the line current. The amount of mobile charge contained at any instant within the elementary segment is $\lambda(V\otimes t)$. where 'V' is the velocity of the charges.

All these mobile charges coming out of segment in ⊗t seconds is called current.

$$I = \frac{\lambda(V\cancel{\otimes}t)}{\cancel{\otimes}t}$$

$$\boxed{\vec{I} = \lambda \vec{V}}$$ where $\lambda = \vec{I} / \vec{V}$ = mobile line charge density

Surface Currents:



Flow of electric charges over a surface represents surface currents. Every surface current is associated with a mobile surface charge density σ c/m^2.

Consider a surface current sheet with mobile surface charge density σ c/m^2 and an elementary rectangle ABCD.

The amount of mobile charges contained at any instant within the elementary rectangle is "$\sigma \otimes l_{\perp}(V\otimes t)$". All these mobile charges within the elementary rectangle coming out in '⊗t' seconds is called current.

$$\otimes I = \frac{\sigma\otimes l_{\perp}(V\cancel{\otimes}t)}{\cancel{\otimes}t}$$

$$\otimes I = \sigma V \otimes l_{\perp}$$

$$\frac{\otimes I}{\otimes l_{\perp}} = \sigma V = K$$

$$\boxed{\vec{K} = \sigma \vec{V}}\ , \text{A/m}$$

where $\vec{K}$ = surface current density, A/m

(Contd …37)

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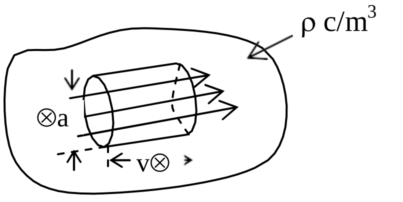
Volume Currents:



Flow of electric charges over a volume represents volume currents. Every volume current is associated with a mobile volume charge density ρ c/m^3. Considering an elementary cylinder within the volume current region, the amount of mobile charges contained at any instant is "$\rho \otimes a_\perp (V \otimes t)$".

All these elementary mobile charges coming out of the elementary cylinder in '$\otimes t$' seconds is called current.

$$\otimes I = \frac{\rho \otimes a_\perp (V \cancel{\otimes t})}{\cancel{\otimes t}}$$

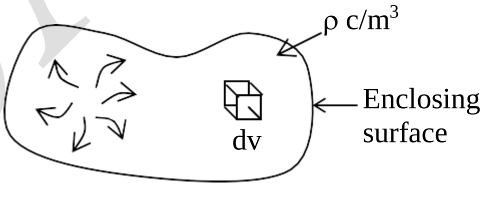
$$\otimes I = \rho V \otimes a_\perp$$

$$\frac{\otimes I}{\otimes a_\perp} = \rho V = J$$

$\therefore$ $\boxed{\vec{J} = \rho \vec{V}}$ A/m^2

Where $\vec{J}$ = Volume current density, A/m^2

Continuity Equation:



Let us consider a region carrying volume currents. For convenience let the charges flow outward. The net outward current through the enclosing surface can be obtained as.

$$\boxed{I = \oint_s \vec{J} \cdot \vec{da}} \quad --- \quad (1) \quad \text{[from volume currents]}$$

And also, the rate of reduction of electric charges within the encloser.

$$= - d/dt \int_V \rho \, dv \quad --- \quad (2)$$

According to the law of conservation of charges the above two equations are equal

$$\oint_s \vec{J} \cdot \vec{da} = - d/dt \int_V \rho \, dv$$

According to the fundamental theorem of divergence

$$\int_V (\nabla \cdot \vec{J}) \, dv = - \partial/\partial t \int_V \rho \, dv \qquad \text{[only one variable]}$$

Integration is done with respect to volume and differentiation is done with respect to time.

(Contd …38)

ECE-SAEC

Therefore $\partial/\partial t$ can be taken inside the integral since the variables are different.

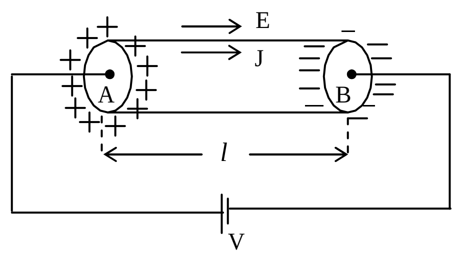
$$\int_V (\nabla \cdot \vec{J})\, dv = - \int_V (\partial\rho / \partial t)\, dv$$

$$\boxed{\nabla \cdot \vec{J} = - \partial\rho / \partial t}$$ ⓪ **Maxwell's 5th equation.**

The above equation is called continuity equation or Fifth Maxwell's equation.
Divergence of J gives net outflow of current per unit volume.
Net overflow of current per unit volume is negative of time rate of charge per unit volume. The above equation is also called as law of conservation of charge.

The above equation explains continuity of current. According to law of conservation of charge, charge can be neither created nor destroyed. Some charge keeps flowing in the circuit. Existing charge cannot be destroyed and new charge cannot be created.

Ohm's Law:



Current flowing through a conductor is directly proportional to the potential difference across it, provided temperature is kept constant.

$$I \propto V$$

The proportionality constant is conductance

$$I = GV$$

$$= V / R$$

$$= \frac{V}{\rho l / A}$$

$$I = VA / \rho l$$

$$\frac{I}{A} = \frac{V}{l} \times \frac{1}{\rho}$$

$$\boxed{\vec{J} = \sigma \vec{E}} \qquad [1/\rho = \sigma]$$

The above equation is called point form or field form of Ohm's law.

Joule's Law:

According to Joules law, whenever current flows through a conductor, heat energy is produced. This is proportional to I^2, R and t.

Heat Energy $\propto I^2Rt$

Energy $\propto (I^2Rt) / J_1$ Where J_1 is called Joule's constant.

We know that power $= I^2R = V^2/R = VI$

(Contd …39)

ECE-SAEC

Multiply and divide with volume

$$P = VI \, . \, Al / Al$$

Rearrange the terms,

$$P = \frac{V}{l} \times \frac{l}{A} (Al)$$

Substitute $E = V/l$, $J = I/A$ and volume = Al

$$\therefore \; P = EJ \text{ volume}$$

EJ can be written as E.J since E and J are in the same direction

$$P = (\vec{E} \, . \, \vec{J}) \text{ volume}$$

$$\therefore \quad \boxed{P = \int_V (\vec{E} \, . \, \vec{J}) \, dv}$$

According to Joule's law, energy dissipated per second is volume integral of dot product of the vectors E and J.

Relaxation Time:

To study relaxation time we start with ohm's law and equation of continuity.

$$\vec{J} = \sigma \vec{E} \text{ and } \nabla \, . \, \vec{J} = - (\partial\rho / \partial t)$$

$$\nabla \, . \, \sigma E = - (\partial\rho / \partial t)$$

$$\nabla \, . \, \frac{\varepsilon}{\varepsilon} \sigma \vec{E} = - (\partial\rho / \partial t)$$

$$\frac{\sigma}{\varepsilon} \nabla \, . \, \vec{D} = - \frac{\partial\rho}{\partial t}$$

$$\frac{\sigma}{\varepsilon} \rho + \frac{\partial\rho}{\partial t} = 0$$

$$\therefore \quad \boxed{\frac{\partial\rho}{\partial t} + \frac{\sigma}{\varepsilon} \rho = 0}$$

$\rho = \rho_0 \, e^{-(\sigma/\varepsilon)t}$ where ρ_0 is charge density at t = 0.

The charge density decays exponentially as time passes with time constant equal to $\in/\sigma$ seconds. This time constant is called relaxation time.

Conductance – Capacitance Theorem:

G = Conductance, C = Capacitance

σ = Conductivity, ε = permittivity ρ = resistivity

According to conductance theorem, conductance of an insulated medium is equal to $\sigma/\in$ times the capacitance of the insulation provided between two conducting media.

$$G = (\sigma / \varepsilon) \, C$$

We know that $C = \in A / l$ and $R = \rho l / A \Rightarrow G = A / \rho l = \sigma A / l$

$$\therefore \; \frac{G}{C} = \frac{\sigma A}{\varepsilon A}$$

$$\boxed{G = (\sigma/\varepsilon) \, C}$$

This theorem is very useful to obtain the expression for conductance of the configuration if capacitance of that configuration is already known. Conductance can be obtained by multiplying capacitance expression with $\sigma/\in$.

(Contd …40)

ECE-SAEC

Observation:

We know that $I = \oint_s \vec{J} \, . \, \vec{da}$

Substitute ohm's law J = σE

$$I = \oint_s \sigma\vec{E} \, . \, \vec{da} \qquad ◎ \qquad (1)$$

According to Gauss law, χ = Q

$$\oint_s \vec{D} . \, \vec{da} = Q$$

$$\oint_s \vec{E} \, . \, \vec{da} = Q / \varepsilon \qquad ◎ \qquad (2)$$

Compare (1) and (2)

$$\boxed{I = \sigma \, . \, (Q / \varepsilon)}$$

Substitute I = V/R and Q = CV

$$\frac{\cancel{V}}{R} = \sigma \, . \, \frac{C\cancel{V}}{\varepsilon}$$

$$\boxed{\frac{1}{R} = G = \frac{\sigma}{\varepsilon} C}$$

Duality:

If two equations are in similar form, they are said to be dual equations.

Duality means that it is possible to pass from one equation to another equation by suitable interchanges of dual quantities.

We know that the conductance of the dielectric between the plates is σA/*l*. Capacitance is εA/*l*. If we know the capacitance of configuration, conductance of that configuration can be obtained by merely replacing ε with σ.

For example capacitance of cylindrical capacitor is

$$C = [(2\pi\varepsilon) / \log(b/a)]$$

Conductance can be obtained by replacing 'ε' with 'σ'.

$$\therefore \; G = [(2\pi\sigma) / \log(b/a)]$$

similarly,

Conductance of spherical capacitor is, $C = (4\pi\varepsilon ab) / (b - a)$

Conductance can be obtained by replacing 'ε' with 'σ'.

$$\therefore \; G = (4\pi\sigma ab) / (b - a)$$

Therefore, ε and σ are dual quantities.

Basic Properties of conductors:

1) Electric field is zero inside a conductor. If there is a field inside, the charges experience a force and they move outwards. Therefore, there is no charge inside.

   Q = 0, D = 0 and E = 0

2) The charges can only reside on the surface of the conductor and not inside a conductor.
3) Conductor is an equipotential region.
4) Electric field intensity at all points on the surface of a conductor must be normal to the surface.
5) Electric charges located outside a conductor cannot produce an electric field inside a completely closed cavity with in the conductor.

(Contd …41)

ECE-SAEC

OBJECTIVES

One Mark Questions

1. The mica layer ($\varepsilon_r = 7$) in a parallel plate capacitor with an effective area of 120mm has a damaged section equivalent to a hole of 0.5mm diameter. Which of the following would be significantly affected by damage. ***(GATE'91,EEE)***
a) capacitance b) charge c) dielectric breakdown d) tan δ

2. Which of the following equations represents the Gauss' law in homogeneous isotropic medium?
a) $\oint\!\!\oint D \,.\, ds = \iiint \rho \, dV$ b) $\vec{\nabla} \times \vec{H} = \vec{D}$ c) $\nabla \,.\, \vec{J} + \rho = 0$ d) $\nabla \,.\, \vec{E} = \rho/\varepsilon$ ***(GATE'92,EEE)***

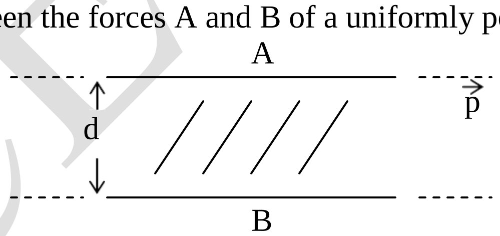
3. The line integral of the vector potential A around the boundary of a surface 'S' represents
a) flux through in the surface S b) flux density in the surface S ***(GATE'93,EEE)***
c) magnetic density d) Current density

4. When a charge is given to a conductor ***(GATE'94,EEE)***
a) it distributes uniformly all over the surface
b) it distributes uniformly all over the volume
c) it distributes on the surface, inversely proportional to the radius of curvature
d) it stays where it was placed

5. Energy stored in a capacitor over a cycle, when excited by an a.c source is ***(GATE'97)***
a) the same as that due to a d.c source of equivalent magnitude
b) half of that due to d.c source of equivalent magnitude
c) zero d) none

6. When the plate area of a parallel plate capacitor is increased keeping the capacitor voltage constant, the force between the plates ***(GATE'99)***
a) increases b) decreases c) remains constant
d) may increase or decrease depending on the metal making up the plates

7. The potential difference between the forces A and B of a uniformly polarized infinite slab shown in figure. ***(IES'93)***



a) $pd / \varepsilon_0(\varepsilon - 1)$
b) $pd / \varepsilon_0 \, \varepsilon$
c) pd / ε_0
d) $pd(\varepsilon + 1) / \varepsilon_0$

8. If $\vec{n}$ is the polarization vector and $\vec{K}$ is the direction of propagation of a plane electromagnetic wave, then ***(IES'93)***
a) $\vec{n} = \vec{K}$ b) $\vec{n} = -\vec{K}$ c) $\vec{n} \,.\, \vec{K} = 0$ d) $\vec{n} \times \vec{K} = 0$

9. Consider the following statements regarding field boundary conditions: ***(IES'95)***
1. The tangential component of electric field is continuous across the boundary between two dielectrics.
2. The tangential component of electric field at a dielectric – conductor boundary is non – zero
3. The discontinuity in the normal component of the flux density at a dielectric conductor boundary is equal to the surface charge density on the conductor.
4. The normal component of the flux density is continuous across the charge free boundary between two dielectrics.

Of these statements
a) 1,2 & 3 are correct b) 2,3 & 4 are correct c) 1,2 & 4 are correct d) 1,3 & 4 are correct

(Contd …42)

ECE-SAEC

10. Consider the following statements associated with a parallel plate capacitor. ***(IES'95)***

1. Capacitor is proportional to area of plates
2. Capacitance is inversely proportional to distance of separation of plates
3. The dielectric material is in a state of compression.

Of these statements

a) 1,2 & 3 are correct b) 1 & 2 are correct c) 1 & 3 are correct d) 2 & 3 are correct

11. Two electric dipoles aligned parallel to each other and having the same axis exert a force F on each other, when a distance 'd' apart. If the dipoles are at a distance '2d' apart, then the mutual force between them would be: ***(IES'95)***

a) F/2 b) F/4 c) F/8 d) F/16

12. When a lossy capacitor with a dielectric of permittivity ε and conductivity σ operates at a frequency ω, the loss tangent for the capacitor is given by ***(IES'95)***

a) $\omega\sigma / \varepsilon$ b) $\omega\varepsilon / \sigma$ c) $\sigma / \omega\varepsilon$ d) $\sigma\omega\varepsilon$

13. The properties of a medium are ***(NTPC'98)***

a) permittivity, permeability, insulation b) permittivity, permeability, conductivity
c) permeability, resistivity, inductivity d) permeability, flux, magnetism

14. The characteristic impedance of a co – axial cable depends on ***(CIVIL SERVICES)***

1. ratio of outer and inner diameter
2. length of the cable
3. logarithmic ratio of outer and inner diameter
4. logarithmic ratio of outer and inner diameter and inversely as the square root of dielectric constant.

The correct statements are

a) 3 & 4 b) 2 & 3 c) 1, 3, 4 d) 4 only

15. The unit of $\mu_0\varepsilon_0$ is ***(NTPC'98)***

a) Farad Henry b) m^2 / sec^2 c) amp sec / volt sec d) Newton $metre^2/coulomb^2$

16. Kirchoff's current law for direct currents is implicit in the expression ***(IES'97)***

a) $\nabla . \vec{D} = f$ b) $\int \vec{J} . \vec{n}\, ds = 0$ c) $\nabla . \vec{B} = 0$ d) $\nabla \times \vec{H} = \vec{J} + \partial\vec{D}/\partial t$

17. Poisson's equation for an inhomogeneous medium is ***(IES'97)***

a) $\varepsilon\nabla^2 V = -\rho$ b) $\nabla . (\varepsilon\nabla V) = -\rho$ c) $\nabla^2(\varepsilon V) = -\rho$ d) $\nabla . (\nabla\varepsilon V) = -\rho$

18. A material is described by the following electrical parameters as a frequency of 10 GHz . $\sigma = 10^6$ mho / m, $\mu = \mu_0$ and $\sigma / \sigma_0 = 10$. The material at this frequency is considered to be ($\sigma_0 = 1/36\pi \times 10^{-9}$ F/m) ***(GATE'93)***

a) a good conductor b) a good dielectric
c) neither a good conductor nor a good dielectric d) a good magnetic material

19. Copper behaves as a ***(GATE'95)***

a) conductor always b) conductor (or) dielectric on the applied electric field strength
c) conductor (or) dielectric depending on the frequency
d) conductor (or) dielectric depending on the electric current density

20. For a dipole antenna ***(GATE'94)***

a) The radiation intensity is maximum along the normal to the dipole axis
b) The current distribution along the length is uniform irrespective of the length
c) The effective length equals its physical length
d) The input impedance is independent of the location of the feed – point

(Contd …43)

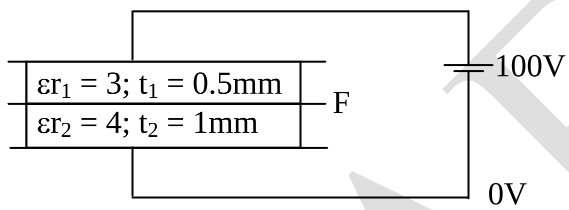
ECE-SAEC

21. The intrinsal impedance of a lossy dielectric medium is given by ***(GATE'95)***
a) $j\omega\mu / \sigma$ b) $j\omega\varepsilon / \mu$ c) $\sqrt{j\omega\mu / (\sigma + j\omega\varepsilon)}$ d) $\sqrt{\mu / \varepsilon}$

22. An antenna, when radiating, has a highly directional radiation pattern. When the antenna is receiving, its radiation pattern ***(GATE'95)***
a) is more directive b) is less directive c) is same d) exhibits no directivity at all

Two Mark Questions

1. A composite parallel capacitor is made up of two different materials with different thickness (t_1 and t_2) as shown. The two different dielectric materials are separated by a conductivity foil F. The voltage of the conductivity foil is. ***(GATE'03, EEE)***
a) 52 V



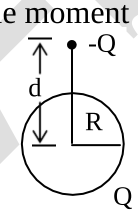
b) 60V

c) 67 V

d) 33 V

2. A parallel plate capacitor has an electrode area of 100 mm^2, with a spacing of 0.1 mm between the electrodes. The dielectric between the plates is air with a permittivity of 8.85×10^{-12} F/m. The charge on the capacitor is 100V. The stored energy in the capacitor is ***(GATE'03, EEE)***
a) 8.85 PJ b) 440 PJ c) 22.1 nJ d) 44.3 nJ

3. A circular ring carrying a uniformly distributed charge Q and a point charges –Q on the axis of the ring are shown. The magnitude of the dipole moment of the charge system is ***(IES'93, EEE)***
a) Qd
b) QR^2 / d
c) $Q\sqrt{R^2 + d^2}$
d) QR



4. Find the polarization in a dielectric material with $\varepsilon_r = 2.8$ if $D = 3 \times 10^{-7}$ c/m^2.
a) 1.93×10^{-7} c/m^2 b) 10^{-19} c/m^2 c) 6.602×10^{-2} c/m^2 d) 0

5. Determine the value of electric field in a dielectric material for which χ is 3.5 and P is 2.3×10^{-7} c/m^2.
a) 7.9×10^{-2} b) 62.1×10^{-3} c) 74.3×10^{2} d) 83×10^{3}

6. Calculate the emerging angle by which the vector E changes its direction as it passes from a medium with $\varepsilon_r = 100$ into air making an angle of 45° with the interface as it enters
a) 90° b) 0.57° c) 0.89° d) 45°

7. Electric flux lines are incident in the porcelain insulator of $\in_r = 6$ at an angle of 45°. The electric field in the insulator is 1000V/m. Determine the electric field in the air and the angle at which flux lines are emerging out
a) 0.46°, 400 V/cm b) 2.25°, 4000 V/cm c) 7.2°, 4925 V/cm d) 9.46°, 4302 V/cm

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Linked Question from Q.No 8 to 11

A parallel plate capacitor consists of two square metal plates of side 500 mm and separated by a 10 mm slab of Teflon with $\varepsilon_r = 2$ and 6mm thickness is placed on the lower plate leaving an air gap of 4mm thick between it and upper plate. A 100V is applied across capacitor.

8. Find the capacitance between the plates
   a) 2.2×10^{-8} F b) 3.16×10^{-10} F c) 4.26×10^{-6} F d) zero

9. Find the electric flux density of Teflon and air
   a) 0.12 $\mu c/m^2$, 0.12 $\mu c/m^2$ b) 0.35 $\mu c/m^2$, 0.12 $\mu c/m^2$ c) 0.11 $\mu c/m^2$, 0.35 $\mu c/m^2$ d) 0 , 0

10. Find the electric field intensity of Teflon and air
    a) 12555 V/m, 6776 V/m b) 13553 V/m, 6776 V/m
    c) 0, 5826 V/m d) 38265 V/m, 38265 V/m

11. Find the electric potential of Teflon and air
    a) 54.21 V, 40.66 V b) 34.11 V, 34.11 V c) 0, 0 d) 1.1 V, 2.4 V

12. Two conducting planes are located at Z equal to '0' and 6 mm. In the region between $0 < Z < 2$ mm there is a perfect dielectric with $\varepsilon r_1 = 2$, for $2 < Z < 5$ mm, $\varepsilon r_2 = 5$. Find the capacitance per square meter of surface if the region for $5 < Z < 6$ mm is filled with air.
    a) 2.8 nF/m^2 b) 3.4 nF/m^2 c) 1.1 nF/m^2 d) 2.2 nF/m^2

13. A 2 μF capacitor is charged by connecting it across 100V D.C supply. The supply is now disconnected and the capacitor is connected in parallel with another uncharged 2μF capacitor. Assuming no leakage of charge, determine the energy stored in capacitor.
    a) 0.01 Joules b) 0.005 Joules c) 1.15 Joules d) 0.5 Joules

14. A parallel plate capacitor with air as dielectric has a plate area of 36π cm^2 and separation of1mm. It is charged to 100V by connecting it across a battery. If the battery is disconnected and distance is increased to 2mm, calculate the energy stored, assuming no leakage of charge
    a) 0.6×10^6 Joules b) 0.2×10^{-4} Joules c) 0.23×10^4 Joules d) 1×10^{-6} Joules

15. A Co – axial capacitor of the compressed gas type is to be designed to have 60×10^{-12} F capacitance and is to work at 200 KV dc. The maximum voltage gradient should not exceed 300KV per cm. If the outside diameter of the inner conductor is 5cm, determine the inner diameter of the outer conductor and length of capacitor. Take the relative permittivity of gas to be 1.0.
    a) 3.1 cm, l = 5m b) 4.2 cm, l = 1m c) 8.3 cm, l = 7m d) 5.7 cm, l = 7m

Key:

One Marks:

1.c 2.a 3.a 4.a 5.c 6.a 7.a 8.c 9.d 10.a 11.d 12.c 13.b 14.d

15.b 16.b 17.a 18.a 19.a 20.a 21.c 22.c

Two Marks:

1.b 2.d 3.a 4.a 5.c 6.b 7.d 8.b 9.a 10.b 11.a 12.b 13.b 14.d

15.d

(Contd …45)

ECE-SAEC

Magnetostatics deals with magnetic field produced by current carrying conductor.

Magnetic field:

A static magnetic field can be produced from a permanent magnet or a current carrying conductor. A steady current of I amperes flowing in a straight conductor produces magnetic field around it. The field exists as concentric circles having centres at the axis of conductor.

If you hold the current carrying conductor by the right hand so that the thumb points the direction of current flow, then the fingers point the direction of magnetic field. The unit of magnetic flux is weber. One weber equals 10^8 maxwells.

Magnetic flux density (B):

The magnetic flux per unit area is called magnetic flux density (or) magnetic induction vector. The unit of B is weber/m^2 (or) Tesla.

The magnetic flux through any surface is the surface integral of the normal component of B. The magnitude and direction of B due to a current carrying conductor is given by 'Biot – savart's law'.

$$B = d\phi / da$$

$$d\phi = B \cdot da$$

$$\boxed{\phi = \iint_S \vec{B} \cdot \vec{da}}$$

Magnetomotive force (M.M.F)

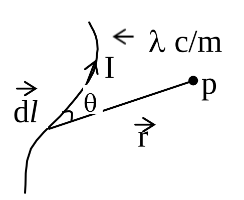
M.M.F is produced when an electric current flows through a coil of several turns. The M.M.F depends on the current and the number of turns. Therefore, the unit for M.M.F is ampere turns. MMF is the cause that produces flux in a magnetic circuit.

Reluctance (s):

Reluctance is the opposition to the establishment of magnetic flux and can be defined as the ratio of M.M.F to the flux produced.

It is directly proportional to the length of the magnetic path and inversely to the cross sectional area of the path. The reciprocal of reluctance is called "PERMEANCE".

Biot – Savart's Law (second Maxwell is equation):



BIOT and SAVART from their experimental observation deduced a mathematical expression for the elementary magnetic flux density produced by a current element at any particular point of observation (p). According to this law considering a current element of length '$\vec{dl}$' carrying a current 'I', the magnetic flux density at a point of observation 'p' is elementary field intensity.

Magnetic field intensity due to entire conductor can be obtained by line integral.

$$H = \int \frac{I\vec{dl} \times \vec{r}}{4\pi |r^3|}$$

$$B = \mu / 4\pi r^3 \int I\vec{dl} \times \vec{r} \qquad [\because B = \mu H]$$

Taking divergence on both sides,

$$\text{div}.B = \mu_0 / 4\pi |r|^3 \int \text{div}\,(I\vec{dl} \times \vec{r})$$

(Contd …46)

ECE-SAEC

we know that $\nabla . (\vec{u} \times \vec{v}) = \vec{v}. \text{curl } \vec{u} - \vec{u}.\text{curl } \vec{v}$

$$\text{div } (\vec{Idl} \times \vec{r}) = \vec{r} . \text{ curl } \vec{Idl} - \vec{Idl}. \text{ curl } \vec{r}$$

$$\nabla.B = \frac{\mu_0}{4\pi|r|^3} \int (\vec{r}. \text{ curl } \vec{Idl} - \vec{Idl}. \text{ curl } \vec{r})$$

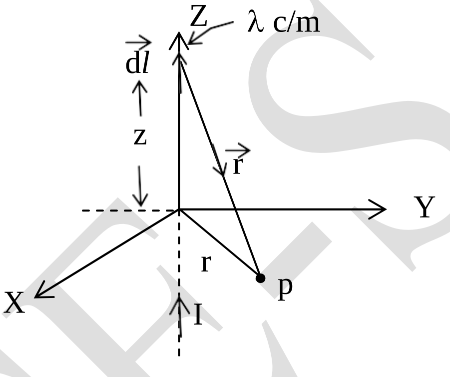
Curl deals with rotation. The current element vector and distance vector have no rotation. Therefore curl of $\vec{Idl}$ and curl of $\vec{r}$ vanish.

$$\nabla.B = \frac{\mu_0}{4\pi|r|^3} \int (0 - 0)$$

$\boxed{\nabla.B = 0}$ © **Maxwell's 2nd equation.**

This equation is called point form, field form, vector form or differential form of BIOT – SAVART law. It is also called second Maxwell's equation.

Magnetic field due to an infinite straight conductor



Consider an infinite straight conductor along the z-axis and carrying a current I along the positive Z-direction. Let 'p' be the point of observation on the x-y plane at a distance 'r' from the z-axis.

Let $\vec{Idl}$ = small current element

We know that,

$$\vec{dB} = \frac{\mu_o}{4\pi} \left(\frac{\vec{Idl} \times \vec{r}}{|r|^3} \right)$$

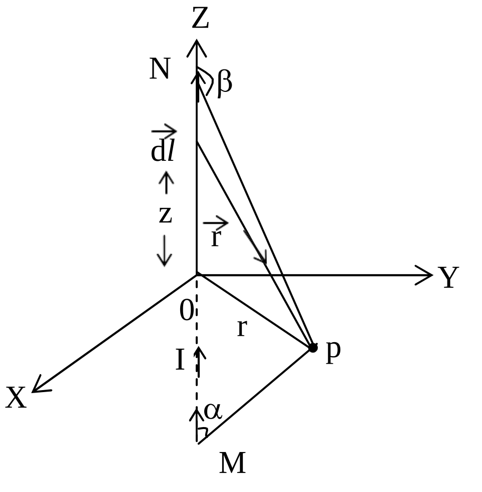
Net magnetic field,

$$\vec{B} = \mu_o /4\pi \int_{-\infty}^{\infty} rdz / (z^2 + r^2)^{3/2} \phi$$

$$\boxed{\vec{B} = \mu_o I /2\pi r \phi}$$

(Contd …47)

Magnetic field due to a finite conductor:



Let us consider a finite conductor of length MN, for the sake of generality OM ≠ ON. Let 'p' be the point of observation on XY plane.

$\therefore$ Net magnetic field $\boxed{\vec{B} = \frac{\mu_0 I}{4\pi r}(\cos\alpha - \cos\beta)\,\hat{\phi}}$

Corollary-1:

Magnetic field due to infinite conductor

i.e $\alpha = 0$, $\beta = 180^o$

$$\therefore \boxed{\vec{B} = \frac{\mu_0 I}{2\pi r}\,.\,\phi}$$

Corollary-2:

Magnetic field due to semi infinite conductor

$\propto = 90^o$, $\beta = 180^o$

$$\boxed{\vec{B} = \frac{\mu_o I}{4\pi r}\,.\,\phi}$$

Corollary-3:

Magnetic field due to finite along the perpendicular bisector

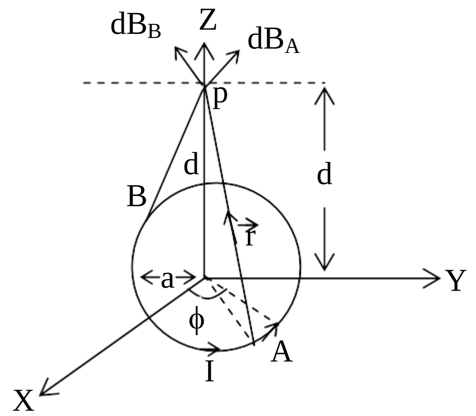
i.e OM = ON

$\alpha = 180 - \beta$

$$\therefore \boxed{\vec{B} = \frac{\mu_o I}{2\pi r}\,.\cos\alpha\,\hat{\phi}}$$

(Contd …48)

Magnetic field due to a circular current carrying loop along its axis:



Consider a circular loop of radius ‘a’ lying on a x-y plane with centre at origin and carrying a current I as shown.

Let the point of observation ‘p’ be at a distance ‘d’ from the centre of the loop. Considering two diametrically opposite current element located at A & B.

Let dB_A & dB_B vectors are corresponding elementary magnetic flux densities at P. Resolving dB_A & dB_B vectors horizontal and vertical components, we find that horizontal components get cancelled and vertical components added up.

∴ Net magnetic field $\boxed{\vec{B} = \frac{\mu_0 I a^2}{2(a^2+d^2)^{3/2}} \cdot z}$

Corollary-1:

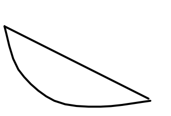
Magnetic field due to circular current carrying loop at its centre i.e d = o.

$$\therefore \boxed{\vec{B} = \frac{\mu_0 I}{2a} \hat{z}}$$

Corollary-2:

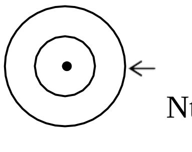
Magnetic field due to a semicircular current carrying loop at its centre

$$\therefore \boxed{\vec{B} = \frac{\mu_0 I}{4a} \hat{z}}$$



Corollary-3:

Magnetic field due to a thin circular coil of ‘N’ turns along the axis



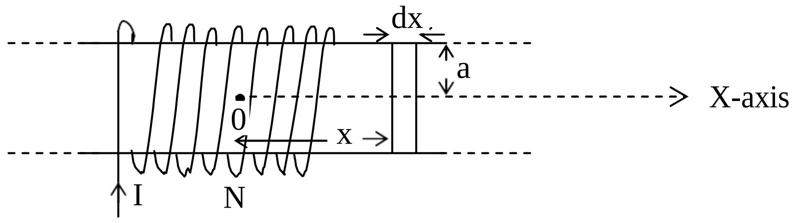
Nturns

$$\boxed{\vec{B} = \frac{\mu_0 N I a^2}{2(a^2+d^2)^{3/2}} \cdot \hat{z}}$$

(Contd …49)

ECE-SAEC

Magnetic field due to an infinite circular solenoidal along its axis:



Let us consider an infinite circular solenoidal of radius ‘a’ with ‘n’ no. of turns per unit length (n=N/l) and carrying a current I. Let the axis of a solenoid coincides with x-axis and origin coincides with the point of observation. Consider an elemental thickness ‘dx’ at a distance ‘x’ from the origin.

Therefore, the elemental magnetic flux density due to this elemental section at point of observation ‘o’ is given by

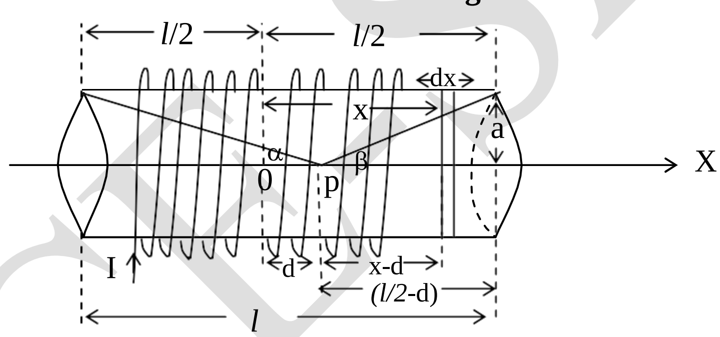
$$dB = \frac{\mu_0 (ndx)\, Ia^2}{2(a^2 + x^2)^{3/2}}$$

$\therefore$ Net magnetic field $\boxed{B = \mu_0 nI}$

The magnetic field due to an infinite circular solenoid is totally confined within the solenoid, uniform and axially directed and is equal to $B = \mu_0 nI$.

The direction of the magnetic field depends on the sense of current carrying by the solenoid and the right hand screw rule.

Magnetic field due to a finite circular solenoid along its axis:



Let us consider a finite circular solenoid of radius ‘a’ and length ‘l’. let ‘n’ be the no. of turns per unit length and ‘I’ be the carrying current. Assume that the solenoid axis coincides with the x-axis and the origin coincides with centre. Let ‘p’ be the point of observation at a distance ‘d’ from the centre.

$$\therefore \quad \boxed{\vec{B} = \frac{\mu_0 nI}{2} (\cos\beta + \cos\alpha)}$$

Corollary-1:

Magnetic field due to an infinite circular solenoid

i.e $\alpha = 0$, $\beta = 0$

$$\therefore \quad \boxed{B = \mu_0 nI}$$

(Contd …50)

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Corollary-2:

Magnetic field due to a finite circular solenoid at the centre
i.e $\alpha = \beta$

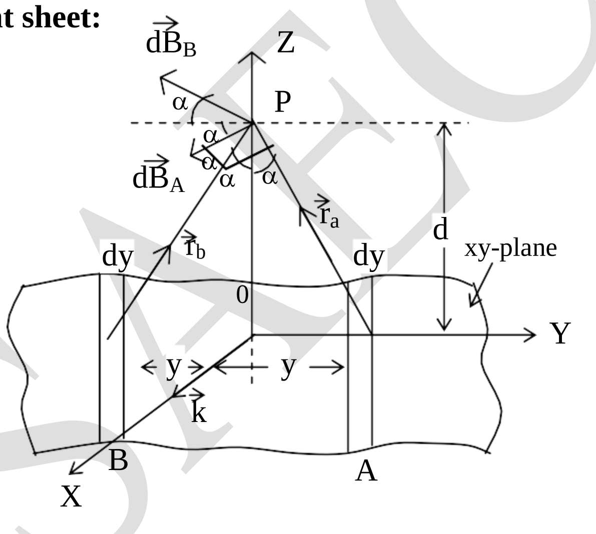
$$\boxed{B = \mu_0 nI \cos\alpha}$$

Corollary-3:

Magnetic field at the end of a finite circular solenoid
$\beta = 90^o$, α

$$\boxed{B = \frac{\mu_0 nI}{2} \cos\alpha}$$

Magnetic field due to an infinite surface current sheet:



Let us consider an infinite current sheet lying on x-y plane carrying a surface current along the positive x-direction ($\vec{K}$) with a surface current density K.

Each strip carries an elementary current dI = kdy.

Net magnetic field $\boxed{\vec{B} = \frac{\mu_0 k}{2} (-\hat{j})}$

It the point of observation is below the surface current sheet, then

$$\boxed{\vec{B} = \frac{\mu_0 k}{2} \hat{j}}$$

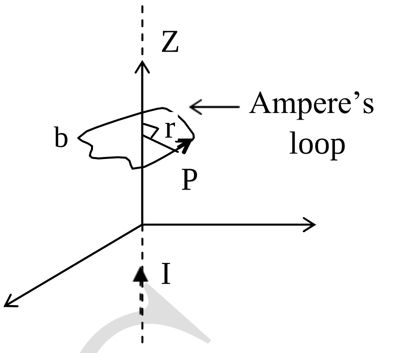
Note:
The magnetic field due to an infinite surface current sheet is independent of the distance of the point of observation from the sheet. The magnetic field due to an infinite sheet is a constant magnitude of $\mu_0 k/2$ and has a direction given by the vector product of $\hat{k} \times \hat{n}$. Where $\hat{n}$ is a unit vector normal to the sheet directed away from the sheet towards the point of observation.

(Contd …51)

ECE-SAEC

TOPIC – 8: AMPERE'S LAW (Ampere's circuital law)

When the magnetic field has some form of symmetry the magnetic flux density can be determined with the application of law known as Ampere's Law.



Consider an infinite straight conductor lying along the Z-axis carrying a current I along the +ve Z-axis. Let 'c' be the closed path enveloping around the conductor. Considering any point 'P' on the closed path, the magnetic flux density at the point 'P' is given by

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\,\hat{\phi}$$

$$\vec{dl} = (dr)\hat{r} + (r\,d\phi)\hat{\phi} + (dz)\hat{z} \quad \text{[cylindrical system]}$$

$$\vec{B}\cdot\vec{dl} = \frac{\mu_0 I}{2\pi r}\; r\,d\phi$$

$$\oint_C \vec{B}\cdot\vec{dl} = \frac{\mu_0 I}{2\pi r}\; r\int_0^{2\pi} d\phi$$

$$= \mu_0 I$$

$$\boxed{\oint_C \vec{B}\cdot\vec{dl} = \mu_0 I_{enclosed}}$$ Ampere's law in integral form

Statement : Considering any closed path in a magnetic field the line integral of tangential component of the magnetic field around the closed path is equal to μ_o times current enclosed.

Differential form of Ampere's Law:

$$\oint \vec{B}\cdot\vec{dl} = \mu_o I_{enclosed}$$

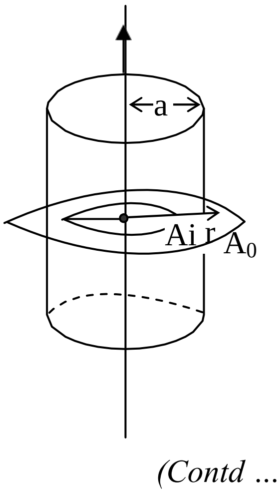
$$\oiint ((\nabla \times \vec{B})\cdot\vec{da} = \mu_o \oiint \vec{J}\cdot\vec{da}$$

$$\boxed{\nabla \times \vec{B} = \mu_o \vec{J}}$$

(or)

$$\boxed{\nabla \times \vec{H} = \vec{J}}$$ **Point from (or) Maxwell's 4th equation**

2. Variation of Magnetic flux density (B) due to a circular conductor:



A solid cylindrical conductor of radius 'a' carries a direct current ' I '.

Inside (r < a) :

Considering the ampere loop A and applying Ampere's Law,

$$\boxed{\vec{B}_i = \frac{\mu_o I r}{2\pi a^2}\,\hat{\phi}}$$

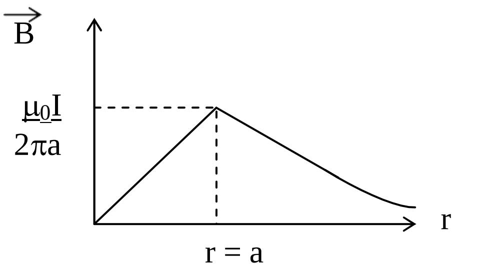
$$\boxed{\therefore B \propto r}$$

(Contd ...52)

ECE-SAEC

Outside (r > a) :

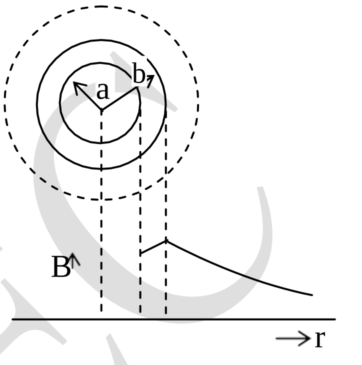
Considering an ampere Loop A_o and applying Ampere's Law,



$$\boxed{\vec{B}_o = \frac{\mu_o I}{2\pi r}\hat{\phi}}$$

$$\boxed{B \; \alpha \; 1/r}$$

3. Variation of Magnetic flux density (B) due to Hallow conductor



Case(i) : (r < a)
Construct an Ampere's Loop such that r < a.
Apply Ampere's Law

$$\boxed{B_i = 0}$$

Case (ii) : (a < r < b)

Construct an ampere's loop and apply Ampere's Law

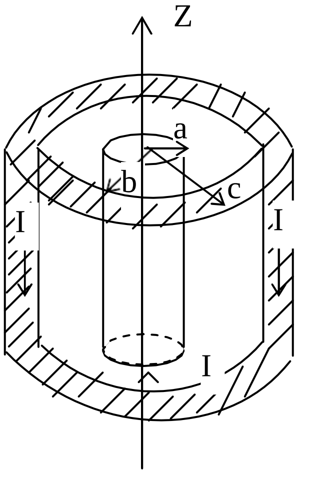
$$\boxed{\vec{B} = \frac{\mu_o I}{2\pi r}\frac{(r^2-a^2)}{(b^2-a^2)}\hat{\phi}}$$

Case(iii) : (r > b)

Construct an ampere's loop and apply Ampere's Law

$$\boxed{\vec{B} = \frac{\mu_o I}{2\pi r}\hat{\phi}}$$

4. Variation of Magnetic flux density (B) due to a pair of coaxial transmission line conductors



Case I: ($0 \le r \le a$)

Considering an ampere loop and applying Ampere's law,

$$\boxed{\vec{B}_1 = \frac{\mu_o I\, r}{2\pi a^2}\hat{\phi}}$$

Case II: ($a \le r \le b$)

$$\boxed{\vec{B}_2 = \frac{\mu_o I}{2\pi r}\hat{\phi}}$$

Case III: ($b \le r \le c$)

$$\boxed{\vec{B}_3 = \frac{\mu_o I}{2\pi r}\frac{(c^2-r^2)}{(c^2-b^2)}\hat{\phi}}$$

Case IV: ($r \ge c$)

$$\boxed{\vec{B}_4 = 0}$$

(Contd …53)

ECE-SAEC

1. Maxwell's Equations for time varying fields:

Differential Form	**Integral Form**
1. $\text{Div } D = \rho$	1. $\int_s D.da = \int_v \rho dv$
2. $\text{Div } B = 0$	2. $\int_s B.da = 0$
3. $\text{Curl } E = -\frac{\partial B}{\partial t}$	3. $\oint E.\, dl = -\frac{\partial}{\partial t}\int_s B.\, da$
4. $\text{Curl } H = J + \frac{\partial D}{\partial t}$	4. $\oint H.\, dl = \int_s J.\, da + \int_s J_d.\, da$
5. $\text{Div } J = -\frac{\partial \rho}{\partial t}$	5. $\int_s J.\, da = -\frac{\partial}{\partial t}\int_v \rho dv$

2. Maxwell's Equations for Static Fields (Time Invariant Fields):

1. $\text{Div } D = \rho$
2. $\text{Div } B = 0$
3. $\text{Curl } E = 0$
4. $\text{Curl } H = J$
5. $\text{Div } J = 0$

3. Maxwell's Equations for Dielectrics:

1. $\text{Div } D = 0$
2. $\text{Div } B = 0$
3. $\text{Curl } E = -(\partial B/\partial t)$
4. $\text{Curl } H = (\partial D/\partial t)$
5. $\text{Div } J = 0$

4. Maxwell's Equations for Good Conductors:

1. $\text{Div } D = 0$
2. $\text{Div } B = 0$

6. $\text{Curl } E = -(\partial B/\partial t)$

3. $\text{Curl } H = J$
4. $\text{Div } J = 0$

5. Maxwell's Equations for Free Space:

1. $\text{Div } D = 0$
2. $\text{Div } B = 0$
3. $\text{Curl } E = -(\partial B/\partial t)$
4. $\text{Curl } H = (\partial D/\partial t)$
5. $\text{Div } J = 0$

(Contd ...54)

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6. Maxwell's Equations for Harmonically Varying Fields:

Substitute $(\partial D/\partial t) = j\omega D$; $(\partial B/\partial t) = j\omega B$

1. Div D = ρ
2. Div B = 0
3. Curl E = $-j\omega B$
4. Curl H = $J + j\omega D = \sigma E + j\omega\varepsilon E = (\sigma+j\varepsilon\omega)E$
5. Div J = $-(\partial\rho/\partial t) = -j\omega\rho$

7. Free Space Electromagnetic Wave Equation:

we know,

Curl E = $-(\partial B/\partial t)$

$\nabla \times E = -\mu\,(\partial H/\partial t)$..(1)

Curl H = $J + (\partial D/\partial t)$

$\nabla \times H = (\partial D/\partial t)$ since J = 0 in free space

$\therefore$ Curl H = $\varepsilon\,(\partial E/\partial t)$ ....(2)

Taking curl on both sides for equation (2)

$\nabla \times \nabla \times H = \varepsilon\,\partial/\partial t\,(\nabla \times E)$

$\nabla(\nabla . H) - \nabla^2 H = \varepsilon\,\partial/\partial t\,(\nabla \times E)$....................(3)

we know that $\nabla . H = 0$ and substitute (1) in (3)

$-\nabla^2 H = \varepsilon\,\partial/\partial t\,(-\mu\,\partial H/\partial t)$

$\nabla^2 H = \mu\varepsilon\,\partial^2 H/\partial t^2$

in free space $\varepsilon_r = 1$, $\mu_r = 1$

$\therefore$ $\boxed{\nabla^2 H = \mu_0\,\varepsilon_0\,\partial^2 H/\partial t^2}$ .....(4)

This is called free space electromagnetic wave equation in terms of 'H'.

From equation 1: $\nabla \times E = -\mu\,(\partial H/\partial t)$

taking curl on both sides and substitute $\nabla \times H = \partial D/\partial t$

$\nabla(\nabla . E) - \nabla^2 E = -\mu\,\partial/\partial t\,(\partial D/\partial t)$

We know that $\nabla . E = 0$

$-\nabla^2 E = -\mu\,\partial/\partial t\,(\varepsilon\,\partial E/\partial t)$

$\therefore$ $\boxed{\nabla^2 E = \mu_0\,\varepsilon_0\,\partial^2 E/\partial t^2}$ .....(5)

since in free space $\varepsilon_r = 1$, $\mu_r = 1$

This is called free space electromagnetic wave equation in terms of 'E'. *(Contd,..55)*

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OBJECTIVES

One Mark Questions

1. Magnetic flux density at a point distance R due to an infinitely long linear conductor carrying a current I is given by ***(CIVIL SERVICES'93)***
(a) $B = 1/(2\mu\pi R)$ (b) $B = \mu I / 2R$ (c) $B = \mu I / 2\pi R$ (d) $B = \mu I / 2\pi R^2$

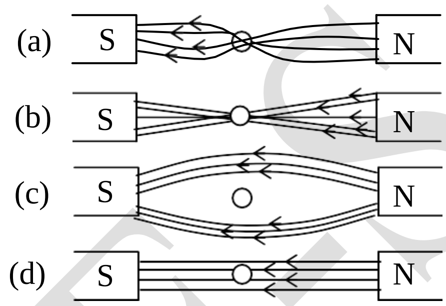
2. Maxwell's divergence equation for the magnetic field is given by ***(CIVIL SERVICES'93)***
(a) $\nabla \times B = 0$ (b) $\nabla . B = 0$ (c) $\nabla \times B = \rho$ (d) $\nabla . B = \rho$

3. Consider the following statements regarding Maxwell's equation in differential form (symbols have the usual meanings) ***(CIVIL SERVICES'94)***

1. For free space $\nabla \times H = (\sigma + j\omega\varepsilon)E$
2. For free space $\nabla. B = \rho$
3. For steady current $\nabla \times H = J$
4. For static electric field $\nabla . D = \rho$

Of these statements:
(a) 1 & 2 are correct (b) 2 & 3 are correct (c) 3 & 4 are correct (d) 1 & 4 are correct

4. When an iron core is placed between the poles of a permanent magnet as shown below, the magnetic field pattern is:



5. The M.K.S unit of magnetic field H is
(a) ampere (b) weber (c) weber per square meter (d) ampere per meter

6. The reflection coefficient, characteristic impedance and load impedance of a transmission line are connected together by the relation
(a) $K_r = \dfrac{Z_L + Z_0}{Z_0 - Z_L}$ (b) $K_r = \dfrac{Z_0 Z_L}{Z_0 - Z_L}$ (c) $K_r = \dfrac{Z_L - Z_0}{Z_L + Z_0}$ (d) $K_r = \dfrac{Z_L - Z_0}{Z_0 Z_L}$

7. The characteristic impedance of a lossless transmission line is given by
(a) $\sqrt{(LC)}$ (b) $\sqrt{(L/C)}$ (c) $1 / \sqrt{LC}$ (d) $\sqrt{(C/L)}$

8. Poynting vector signifies
(a) current density vector producing electrostatic field
(b) power density vector producing electromagnetic field
(c) current density vector producing electromagnetic field
(d) power density vector producing electrostatic field

9. The capacitance per unit length and the characteristic impedance of a lossless transmission line are 'C' and 'Z_0' respectively. The velocity of a traveling wave on the transmission line is: ***(GATE'96)***
(a) Z_0C (b) $1 / (Z_0C)$ (c) Z_0 / C (d) C / Z_0

10. The equation for distortionless transmission is R/G = L/C. To attain it, in a line,
(a) of all the parameters, it is best to increase L for distortionless transmission
(b) Keeping R, and L constant it is preferable to increase or decrease G, and C
(c) the inductance can be added at any interval
(d) the inductance can be of any value

(Contd ...56)

ECESAEC

11. The inconsistency of continuity equation for time varying fields was corrected by Maxwell and the correction applied was ***(CIVIL SERVICES)***
(a) Ampere's law, $\partial D/\partial t$
(b) Gauss's law, J
(c) Faraday's law, $\partial B/\partial t$
(d) Ampere's law, $\partial \rho / \partial t$

12. Which one of the following statements DOES NOT pertain to the equation $\nabla \cdot B = 0$?
(a) There are no sinks and sources for magnetic fields ***(IES'97)***
(b) Magnetic field is perpendicular to the electric field
(c) single magnetic pole cannot exist
(d) B is solenoidal

13. For incidence from dielectric medium (ε_1) into dielectric medium 2(ε_2) the browster angle θ_p and the corresponding angle of transmission θ_t for $\varepsilon_2/\varepsilon_1 = 3$ will be respectively ***(IES'98)***
(a) 30° and 30°
(b) 30° and 60°
(c) 60° and 30°
(d) 60° and 60°

14. A transmission line whose characteristic impedance is a pure resistance ***(GATE'92)***
(a) must be a lossless line
(b) must be a distortionless line
(c) may not be a lossless line
(d) may not be a distortionless line

15. A very lossy, $\lambda/4$ long, 50 ohms transmission line is open circuited and the load end. The input impedance measured at the other end of the line is approximately ***(GATE'97)***
(a) 0
(b) ∞
(c) 50 ohms
(d) none of the above

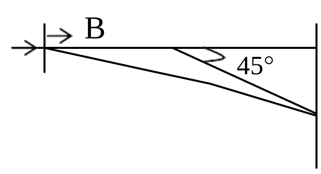
16. The intrinsic impedance of copper at high frequencies is ***(GATE'98)***
(a) purely resistive
(b) purely inductive
(c) complex with a capacitive component
(d) complex with an inductive component

17. The depth of penetrations of wave in a lossy dielectric increases with increasing ***(GATE'98)***
(a) conductivity
(b) permeability
(c) wave length
(d) permittivity

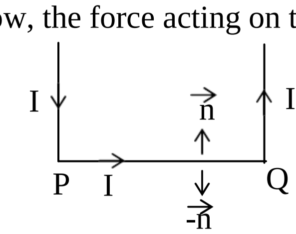
18. The equation $\nabla \cdot J = 0$ is known as ***(IES'00)***
(a) Poisson's equation
(b) Laplace equation
(c) Continuity equation
(d) Maxwell equation

Two Mark Questions

19. A slab of uniform magnetic field deflects a moving charged particle by 45° as shown in figure. The kinetic energy of the charged particle at the entry and exit points in the magnetic field will change in the ratio of
(a) $1 : \sqrt{2}$
(b) $\sqrt{2} : 1$
(c) 1 : 1
(d) 1 : 2



20. In the figure shown below, the force acting on the conductor PQ is in the direction of
(a) PQ
(b) QP
(c) – n
(d) n



(Contd ...56)

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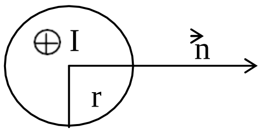
21. A straight wire of circular cross – section carries a direct current I, as shown in figure below. If R is the resistance per unit length of the wire, the poynting vector at the surface of the wire will be ***(IES'93)***

(a) $\frac{RI^2}{2\pi r}\cdot\vec{n}$ (b) $\frac{RI^2}{2\pi r}\cdot(-\vec{n})$



(c) $\frac{RI^2}{2\pi}\cdot\vec{n}$ (d) $\frac{RI^2}{2\pi}\cdot(-\vec{n})$

22. A transverse electromagnetic wave with circular polarization is received by a dipole antenna. Due to polarization mismatch, the power transfer efficiency from the wave to the antenna is reduced to about ***(GATE'96)***

(a) 50% (b) 33.3% (c) 25% (d) 0%

23. Following equations hold for the time – varying fields: ***(ICS'96)***

i) $\nabla \times E = -(\partial B/\partial t)$
ii) $E = -\nabla V - (\partial A/\partial t)$
iii) $\nabla^2 V + \partial/\partial t(\nabla . A) = -(\rho_v/\varepsilon)$
iv) $\vec{B} = \nabla \times \vec{A}$
v) $\nabla \times \vec{H} = \vec{J} + \partial E/\partial t$

In the above equation:

(a) both V and $\vec{A}$ are completely defined and thus can be evaluated
(b) V is completely defined but not $\vec{A}$
(c) $\vec{A}$ is completely defined but not V (d) both $\vec{A}$ and V are not completely defined

24. Match List – I with List – II and select the correct answer using the codes given below the lists: ***(ICS'96)***

List – I	List – II
A) $\oint (J + \partial D/\partial t) . n\, ds$	1) zero
B) $-\oint_s (\partial B/\partial t) . n\, ds$	2) $\oint_v dv$
C) $\oint_s D . n\, ds$	3) $\oint_c E . dl$
D) $\oint_s B . n\, ds$	4) $\oint_c H . dl$
	5) $\oint_v B\, dv$

Codes:

(a) A-4,B-3,C-2,D-1 (b) A-3,B-4,C-2,D-1 (c) A-2,B-5,C-4,D-1 (d) A-4,B-2,C-3,D-1

25. The energy stored in the magnetic field of a solenoid 30 cm long and 3 cm diameter wound with 1000 turns of wire carrying a current of 10A is ***(GATE'96)***

(a) 0.015 Joule (b) 0.15 Joule (c) 0.5 Joule (d) 1.15 Joule

26. Match List – I with List – II and select the correct answer using the codes given below the lists: ***(IES'95)***

List – I(Maxwell's equation)	List – II
A) $\nabla \times H = J + \partial D/\partial t$	1) Faraday's law
B) $\nabla \times E = -(\partial B/\partial t)$	2) Gauss's Law
C) $\nabla . D = \rho$	3) Ampere's law

Codes:

(a) A – 3,B – 1,C – 2 (b) A – 2,B – 1,C – 3 (c) A – 3,B – 2,C – 1 (d) A – 1,B – 2,C – 3

27. Match List – I with List – II and select the correct answer using the codes given below the lists: ***(ICS)***

List – I	List – II
A) $\nabla \times E = 0$	1) $\oint H . dl = \int J . dA$
B) $\nabla . D = \rho$	2) $\oint E . dl = 0$
C) $\nabla \times B = \mu_0 J$	3) $\oint B . dl = 0$
D) $\nabla . B = 0$	4) $\oint E . dA = \int \rho\, dV$

Codes:

(a) A-1,B-2,C-3,D-4 (b) A-2,B-3,C-1,D-4 (c) A-3,B-4,C-1,D-2 (d) A-2,B-4,C-1,D-3

(Contd …58)

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28. Match List – I with List – II and select the correct answer using the codes given below the lists: ***(ICS)***

List – I	List – II
A) Electric field E	1) amp/metre^2
B) Magnetic flux density B	2) coulomb/metre^2
C) Current density J	3) amp/metre
D) Magnetic field strength H	4) Volt/metre
	5) Tesla

Codes:
(a) A-5,B-4,C-1,D-2 (b) A-4,B-3,C-2,D-1 (c) A-1,B-4,C-2,D-5 (d) A-4,B-5,C-1,D-3

29. A transmission line of characteristic impedance 300∧ is terminated by a load of (300 – j300)∧. The transmission coefficient is ***(NTPC'98)***
(a) 1.12 ≤76.68° (b) 1.08 ≤76.68° (c) 1.265 ≤-18.43° (d) 0.791≤-18.45°

30. The input impedance of a lossless transmission line is 100∧ when terminated in a short – circuit, and 64∧ when terminated in an open circuit. The characteristic impedance of the line is
(a) 80∧ (b) 164∧ (c) 36∧ (d) 64∧ ***(IES'97)***

31. Match List – I with List – II and select the correct answer using the codes given below the lists: ***(IES'98)***

List – I	List – II
A) $\nabla \cdot D = \rho$	1) Ampere's law
B) $\nabla \cdot J = -(\partial\rho/\partial t)$	2) Gauss's law
B) $\nabla \times H = J_C$	3) Faraday's Law
C) $\nabla \times E = -(\partial B/\partial t)$	4) Continuity equation

Codes:
(a) A-4,B-2,C-1,D-3 (b) A-2,B-4,C-1,D-3 (c) A-4,B-2,C-3,D-1 (d) A-2,B-4,C-3,D-1

32. Which of the following pairs of parameters and expressions is/are correctly matched? ***(IES'98)***
1. Characteristic impedance ……… $(E/H)\sqrt{\varepsilon_r}$
2. Power flow density ....$\nabla \times H$
3. Displacement current in non - conducting medium ....$E \times H$

Select the correct answer using the codes given below.
Codes:
(a) 1 alone (b) 2 and 3 (c) 1 and 3 (d) 1 and 2

33. If the electric field $E = 0.1te^{-1}a_x$ and $\varepsilon = 4\varepsilon_0$, then the displacement current crossing an area of $0.1m^2$ at t = 0 will be ***(IES'98)***
(a) zero (b) 0.04 ε_0 (c) 0.4 ε_0 (d) 4ε_0

34. The wave length of a wave with propagation constant $(0.1\pi + j0.2\pi)m^{-1}$ is ***(GATE'98)***
(a) $2/\sqrt{0.05}$m (b) 10m (c) 20m (d) 30m

35. The polarization of wave with electric field vector $E = E_0\, e^{j(\omega t + \beta z)}\,(a_x + a_y)$ is ***(GATE'98)***
(a) Linear (b) elliptical (c) left hand circular (d) right hand circular

36. The vector H in the far field of an antenna satisfies ***(GATE'98)***
(a) $\nabla \cdot H = 0$ and $\nabla \times H = 0$ (b) $\nabla \cdot H \neq 0$ and $\nabla \times H \neq 0$
(c) $\nabla \cdot H = 0$ and $\nabla \times H \neq 0$ (d) $\nabla \cdot H \neq 0$ and $\nabla \times H = 0$

Key

1.c	2.b	3.c	4.c	5.b	6.c	7.b	8.b	9.b	10.a	11.a	12.b	13.c
14.c	15.a	16.d	17.c	18.b	19.c	20.c	21.b	22.a	23. a	24.a	25.b	26.a
27.d	28.d	29.c	30.a	31.b	32.a	33.b	34.b	35.a	36.c			

(Contd…..59)

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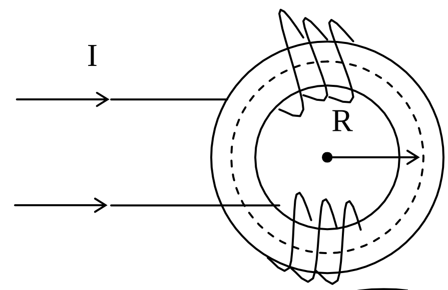
TOPIC – 10: INDUCTANCE OF SIMPLE GEOMETRIES E M F

1. INDUCTANCE OF A TOROIDAL COIL:

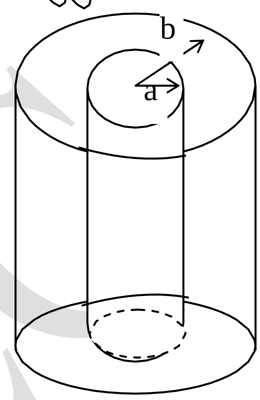


R is Mean radius and N is No. of turns

$$\boxed{\therefore\ L = \frac{\mu_o N^2 S}{2\pi R}}$$

where S = area of cross-section of the core

2. INDUCTANCE OF A COAXIAL CABLE:

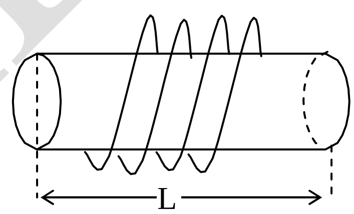


$$\boxed{\therefore\ L = \frac{\mu_o\, ln(b/a)}{2\pi} \quad \text{H/m}}$$

Total inductance of the cable can be obtained by multiplying the above equation with the length of the cable.

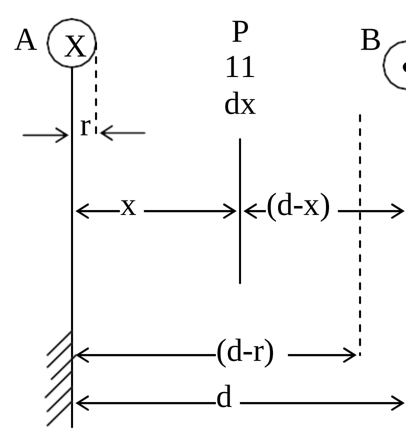
3. INDUCTANCE OF SOLENOID:

$$\therefore\ \boxed{L = \frac{N^2 A\mu}{l}}$$



4. INDUCTANCE OF 1-ϕ LINE ;-

$$\boxed{L = 2ln\left(\frac{d-r}{r}\right) \times 10^{-7}} \quad \text{H/m / conductor}$$

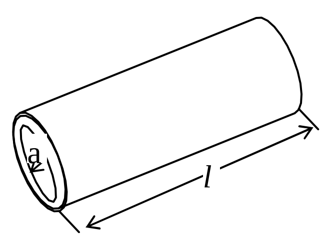


For a transmission line of length 'l' meter, there are l number of inductors in series.Total inductance is the product of inductance per meter length and the length of the line.

Total inductance =L l

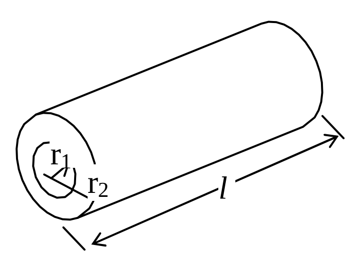
Loop inductance is a series combination of forward and return conductors. Loop inductance of single phase line is 2Ll .

5. SINGLE LAYER AIR CORE COIL:



$$\boxed{\therefore\ L = \frac{39.5\ N^2 a^2}{9a+10l}}$$

6. MULTI LAYER AIR CORE COIL:



$$\boxed{\therefore\ L = \frac{31.6\ N^2 r_1}{6r_1 + 9l + 10(r_2 - r_1)}}$$

** ALL THE BEST **

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